



**High School  
Honors Physics 1st  
Packet 1  
Eckstrom**

**4TH QUARTER  
CURRICULUM PACKET**

**Hayward Community  
School District  
715-634-2619**

**#HurricaneStrong**



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Office Hours: 10:00 am - noon

Monday - Friday



- Week 1
  - Review by doing the CVPM reading
  - Do CVPM Practice and Exploration #4
  - Do CVPM Practice and Exploration #5
  - Optional if you have a computer and internet:
    - Read vpython project notes
    - Do vpython lesson 1: intro
    - Do vpython lesson 2: loops, physics and graphs
    - Do vpython lesson 2b: CVPM scenarios.
- Week 2
  - Do CVPM Unit Review
  - Start CAPM reading--Stop at "Enhanced Equations for Describing Motions"
  - Do CAPM Model-Builder #1
  - Do CAPM Model-Builder #2
- Week 3
  - Finish CAPM reading
  - Do CAPM Practice and Exploration #1
  - Do CAPM Practice and Exploration #2a
  - Do CAPM Practice and Exploration #2b
- Week 4
  - Do "Stacks of Kinematic Graphs" practice set
  - Do CAPM Practice and Exploration #3
  - Do CAPM Practice and Exploration #4
- Week 5
  - Do CAPM Unit review
  - Optional if you have a computer and internet:
    - Vpython Lesson 3: CAPM and model accuracy
  - Start BFPM reading--stop after "Free-Body Diagrams"
  - Do Introduction to Forces Model-Builder
  - Do BFPM Practice and Exploration #1
- Week 6
  - Do BFPM Practice and Exploration #9-2
  - Do BFPM Practice and Exploration #9-3
  - Do BFPM Practice and Exploration #4

## Constant Velocity Particle Model: The simplest model of motion in physics.

As you are reading this . . .

- *If you have the capability to access this document on the class Moodle page, your experience will be better. If you can't, you can still get a lot out of it, but obviously you will not be able to access the check quizzes and other links.*
- *you will see some words that are bold and in italics. These are words that will be important to look up if you don't understand them. Some are links to dictionary definitions, but if that doesn't clear it up for you, do some exploring until you think you get what the word means.*
- *you will see "check for understanding" links. I've put them there to help you see if you're picking up what was in the section you've just finished. This reading is really of almost no value if you just keep reading through it without understanding, so use these little quizzes to check your understanding. If you find that you are mostly answering the questions incorrectly, it would probably be a good idea to go back, re-read and take the check. Repeat if necessary.*

### Introduction:

One of the main focuses in physics is using models to describe how things move. This reading activity is a first look at how physics models the simplest type of motion there is, so we are going to restrict ourselves to . . .

- One dimension. This means there is only forward and back. No right and left. No up and down. Dealing with motion in multiple directions gets really complicated really fast, so we won't cover it here.
- Constant velocity. This means the objects whose motion we'll be modeling will not be speeding up or slowing down. Models for that kind of motion exist, but it's best for now to not deal with them at first, because they are much easier to understand after you understand constant velocity motion models.

In this reading, we are going to develop and examine four representations of motion . . .

- Graphs of position (place) and time.
- Maps of position during a motion.
- Graphs of velocity (kind of like speed) and time.
- Symbolic representations (equations) of motion.



Check your understanding now by clicking on the check mark:

### Graphs of position and time:

Imagine a fancy elevated high-speed commuter train moving between two fictional cities, Happyville and Busytropolis. There are many different ways that this motion could be described. As you have probably figured out if you've been in a physics class for a while, the ones that scientists are most interested in will probably involve measurement data. Here's a possible experiment to create some data:

- The independent variable will be the time the train has been moving since it left



Happyville.

- The dependent variable will be the position of the train, as measured by the kilometer



markers along the side of the track.

- The operator of the train will control the throttle setting of the train so it doesn't speed up or slow down during the part of the trip we are measuring.

Here's the data:



- The clock starts just as the train passes the 12 km marker:

Time (minutes):	0
Position (km):	12



- When the clock reads 12 minutes, the train is passing the 36 km marker:

Time (minutes):	0	12
Position (km):	12	36



- When the clock reads 18 minutes, the train is passing the 48 km marker:

Time (minutes):	0	12	18
Position (km):	12	36	48



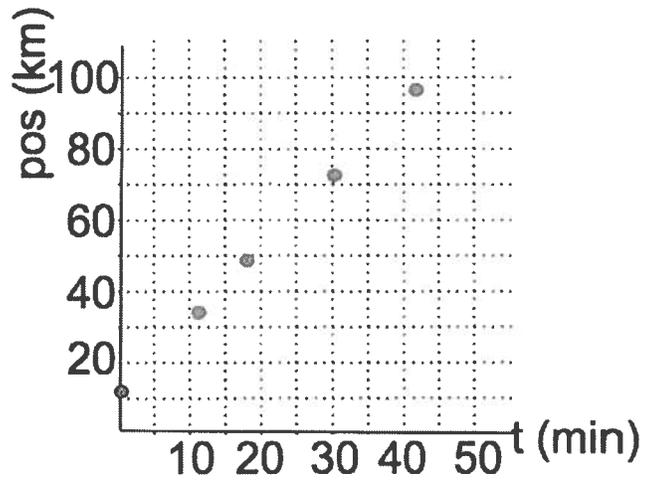
- When the clock reads 30 minutes, the train is passing the 72 km marker:

Time (minutes):	0	12	18	30
Position (km):	12	36	48	72



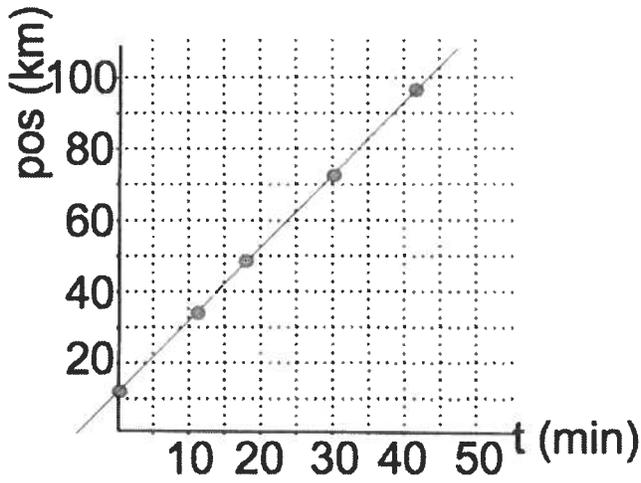
- When the clock reads 42 minutes, the train is passing the 96 km marker:

Time (minutes):	0	12	18	30	42
Position (km):	12	36	48	72	96

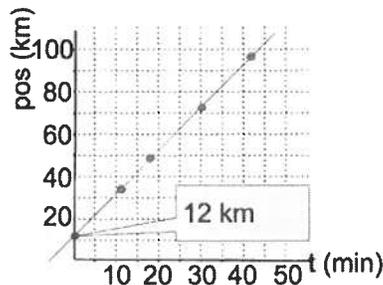


Now let's plot the data as a scatter plot:

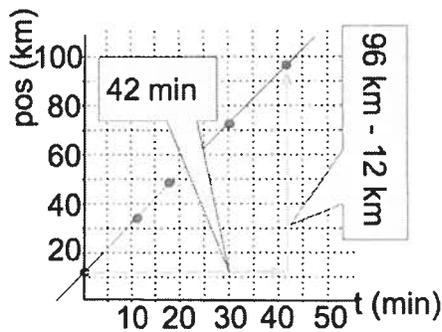
It's reasonable to think that a line is a good model for the motion of this train:



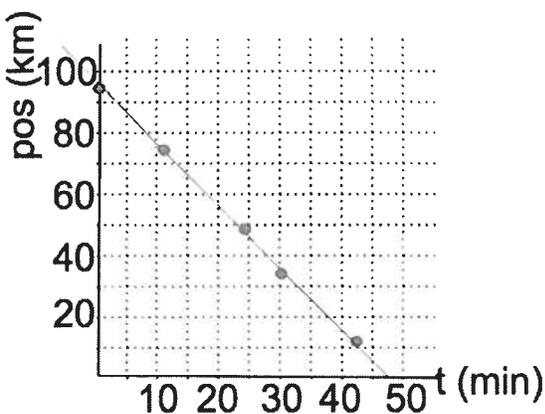
Let's look at the features of this line:



- The vertical intercept is 12 km. This means that the position of the train was 12 km from Happyville when the clock started.



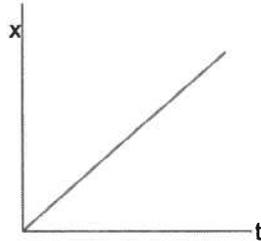
- The slope is  $\frac{96 \text{ km} - 12 \text{ km}}{42 \text{ min}} = 2 \frac{\text{km}}{\text{min}}$ . This represents the speed of the train.
- If the train had been moving toward Happyville, it's not hard to imagine what the graph of position vs. time would look like:



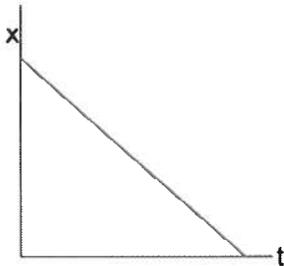
- In this case, the starting position would be 96 km.
- The train would still have a speed of  $2 \frac{\text{km}}{\text{min}}$ , but the slope would not be; it would be  $-2 \frac{\text{km}}{\text{min}}$ .
- Let's define this graphical model the way physicists think about this . . .
  - The vertical axis tracks the position of an object. Position is usually given the name "x" for one-dimensional motion.
    - The vertical intercept of a x vs. t graph is the starting position of the object. This is often called  $x_i$  (for initial time) or  $x_0$  (for position at time zero).
    - How the object's position changes is called its **displacement**. This usually designated by " $\Delta x$ ", because " $\Delta$ " (pronounced "delta") is a symbol that means "change in".
      - Positive displacement means the object moved in the forward direction (like the train in the first graph).
      - Negative displacement means the object moved in the backward direction (like the train in the second graph).
      - **Distance** is the word we use to mean how far the object moved. (The trains moved the same distance in both graphs above, but they didn't have the same displacement. Why?)

- The slope of a  $x$  vs.  $t$  graph is called the **velocity**. It represents how fast the object's position is changing.

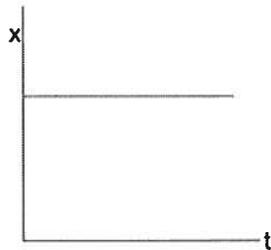
- A positive slope means the object is traveling forward.



- A negative slope means the object is traveling backward.



- A zero slope (horizontal line) means the object is not moving.



- The steepness of the slope is the object's **speed**.

- This is a graph of a straight line, which means that the object's velocity is not changing over time. We will call this model of motion the **Constant Velocity Particle Model (CVPM)**.
- The graphs of lines all have an equation that fits the pattern of  $dependent = intercept + slope \cdot independent$ . So for CVPM motion, the algebraic model is  $position = initial\ position + velocity \cdot time$  or for short  $x = x_i + v \cdot t$ 
  - The train in the first graph could be modeled as  $x = 12\ km + (2\ \frac{km}{min}) \cdot t$ .
  - The train in the second graph could be modeled as  $x = 96\ km + (-2\ \frac{km}{min}) \cdot t$ .

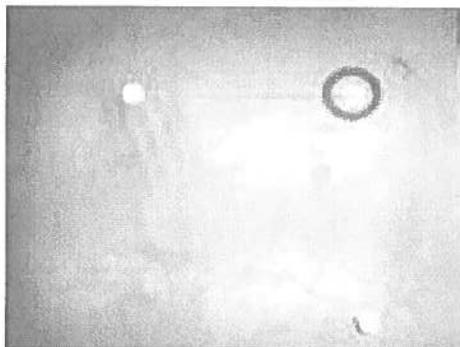


Check your understanding now by clicking on the check mark:

**Motion Maps:**

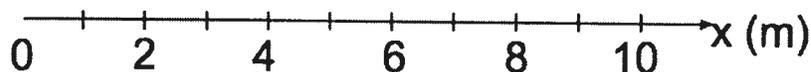
Graphs of position vs. time are a very useful way to represent a motion. As we've just seen, they lead to a great mathematical model, because graphs can be represented by equations quite nicely. But for some problem-solving strategies, it might be nice to have something that's a little more like a picture of the actual motion itself. This representation is called a motion map.

A good way to get a quick feel for what a motion map consists of might be to watch the video below. If you are reading this on paper, here's the link: <https://youtu.be/rfPfXJV4rTg>

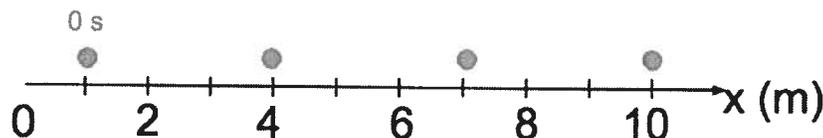


Motion maps have the following features:

- There is some indication of the scale of position and the direction. Since this is one-dimensional motion, a simple number line will suffice:

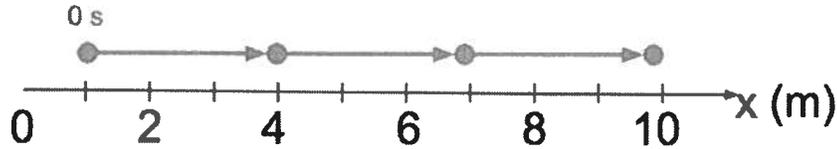


- Dots are placed along this scale that represent the position of the object at equal time intervals (usually each second, but there could be exceptions).
- It's helpful to know where time starts, so usually at least the first second is given a label.
  - For example, if an object is moving forward, starting at 1 meter and moving at 3 m/s, the first dot would be placed at 1 m and each dot after that would be 3 meters forward from the previous dot:

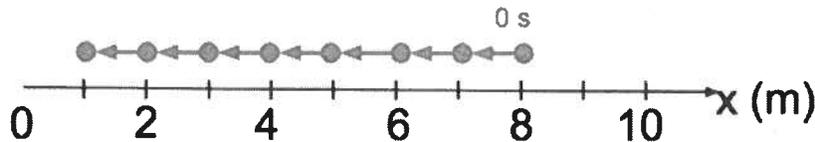


- The last step in creating a good motion map is indicating the direction and size of the velocity at each second using a vector, which is just an arrow that is used to model quantities like velocity that have both size and direction.
  - In constant velocity motion, the vector lengths and directions will all be the same.
  - A good way to make sure your vectors are a good model is to extend each dot's vector to the next dot. For our example from above, the vectors would be 3 units

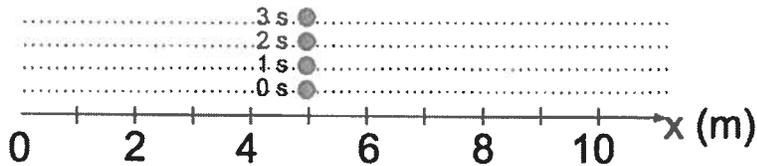
long (representing a speed of 3 m/s) and pointing forward (representing positive direction.)



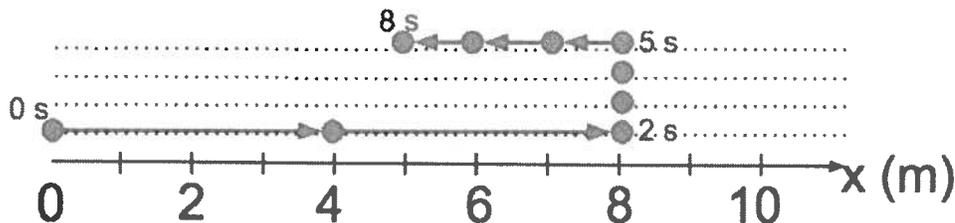
- Here are three additional notes about making motion maps:
  - If the velocity is negative, the velocity vectors simply point backwards. For example, the motion map for an object that started at 8 m and had a velocity of -2 m/s would look like this:



- If the velocity is zero (the object is not moving), we show that time continues to flow by stacking dots above each other in the same spot on the position scale, with no velocity vectors. For example, an object that stayed stationary at 5 m for 3 seconds could be mapped this way:



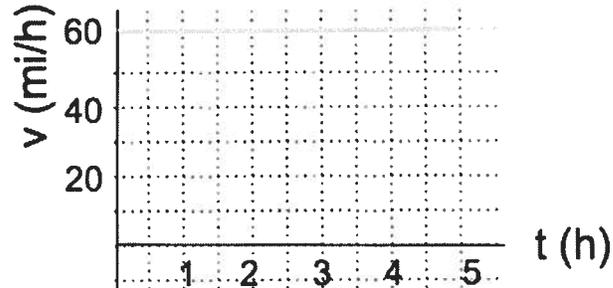
- Objects that do several different kinds of constant velocity motion one after the other can be modeled, as well. For example an object that went forward from position zero at 4 m/s for 2 seconds, then stopped for 3 seconds, then went backwards at 1 m/s for 3 seconds, could be mapped like this:



Check your understanding now by clicking on the check mark:

### Graphs of velocity and time:

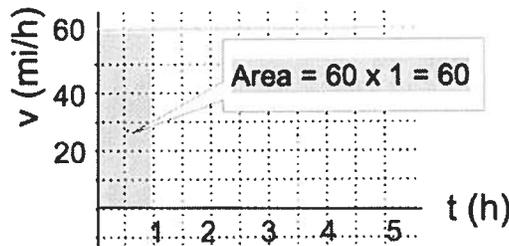
Imagine a car driving on a freeway with the cruise control set at 60 miles per hour. If we were to make a graph of the speedometer reading every 15 minutes, it would be a pretty boring graph,



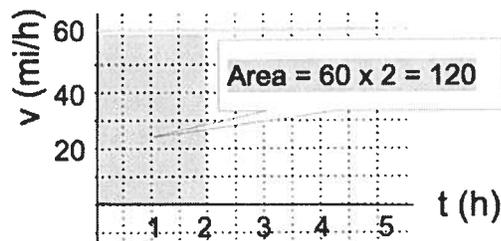
because the speed never changed.

But there is a useful thing about this graph that makes it worth examining for a bit. If someone asked you how far a car would travel if it went 60 miles per hour for an hour, you would probably say, "Well, obviously, it went 60 miles." How far it would go in two hours is not much harder . . . 120 miles. A half hour? . . . you probably know that answer, too.

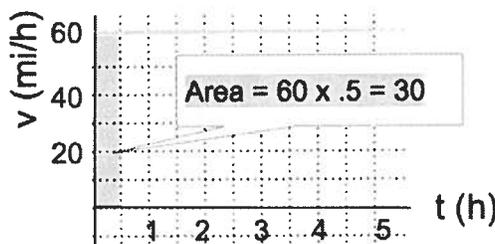
Let's go back to the graph and see where the calculations you made in your head can be seen from the graph.



- One hour (60 miles):



- Two hours (120 miles):



- Half hour (30 miles):

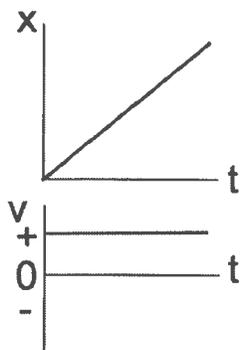
See the pattern? The area under the graph of velocity vs. time is the displacement.

This idea seems a bit silly, because it's so easy. But when we study motions that get more complicated, it becomes a powerful tool. To set up for that, let's explore how to translate  $x$  vs.  $t$  graphs to  $v$  vs.  $t$  graphs.

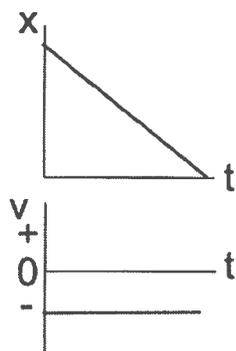
The key to translating a graph of position vs. time to a graph of velocity vs. time is slope. Since velocity is the slope of a position vs. time graph, when you graph  $v$  vs.  $t$ , you are indicating the slope.

When the motion is constant velocity, the lines on a velocity vs. time graph are always horizontal lines. In CVPM, there are only really 3 basic shapes that we have to translate:

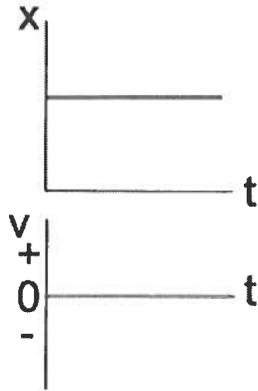
- If the object is moving forward, the slope of  $x$  vs.  $t$  is constant and positive, so the horizontal  $v$  vs.  $t$  line is above the zero line:



- If the object is moving backward, the slope of  $x$  vs.  $t$  is constant and negative, so the horizontal  $v$  vs.  $t$  line is below the zero line:



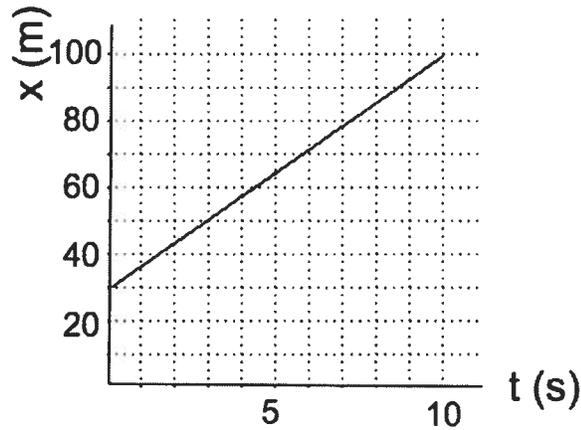
- If the object is not moving, the slope of  $x$  vs.  $t$  is zero, so the horizontal  $v$  vs.  $t$  line is at the zero line:



Check your understanding now by clicking on the check mark:

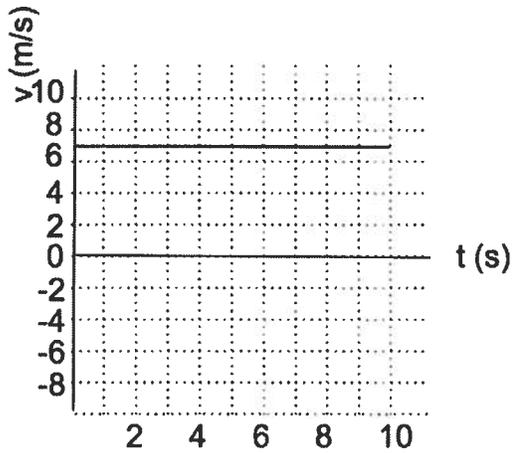
To make the graphs quantitatively accurate, you will have to actually determine the slope. Let's try a couple of examples.

- This graph shows an object that starts at a position of 30 m and moves to a position of

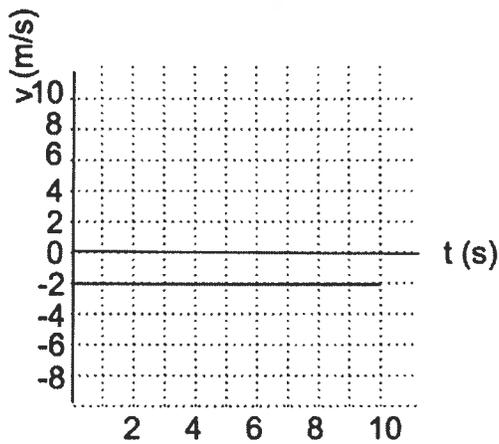


100 m in 10 seconds.

- This makes the slope (velocity)  $\frac{100\text{ m} - 30\text{ m}}{10\text{ s}} = \frac{70\text{ m}}{10\text{ s}} = 7\text{ m/s}$
- To make the velocity vs. time graph, since the velocity is a constant 7 m/s, we draw a constant horizontal line at 7 m/s:

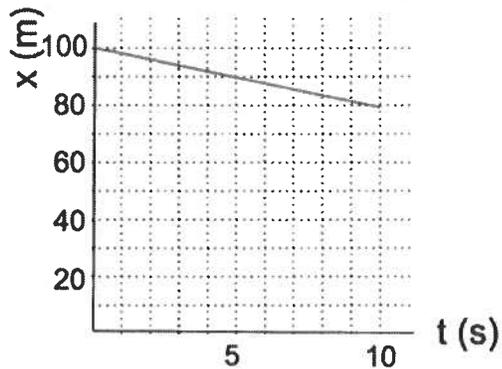


- Let's try one in reverse. This graph represents an object that is moving backwards at 2

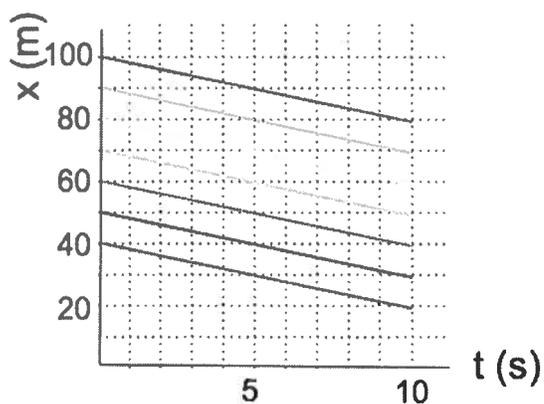


m/s.

- This means that the slope of the  $x$  vs.  $t$  graph must be  $-2$  m/s. This is  $-20$  m in 10 s, so I'll draw an  $x$  vs.  $t$  graph that starts at 100 m and drops to 80 m in 10 s.



- I've picked a starting point of 100 m, but there is nothing on the  $v$  vs.  $t$  graph that tells me where the object actually started. Any  $x$  vs.  $t$  graph that shows a line parallel to the one I drew is equally good.



Check your understanding now by clicking on the check mark:

One last thing before we finish. After you have a little practice with the graphical models, you can often skip straight to the mathematical model directly from a description of the motion. Remember that the pattern the equation will always take is  $x = x_i + v \cdot t$ . Where  $x$  is the position at any time ( $t$ ),  $v$  is the velocity and  $x_i$  is the initial (starting) position.

For example, if a missile is moving toward its target at 500 mi/hr and its position when timing starts is 1250 miles from the target, we can quickly model its position relative to the target (in other words, if the target is position 0, where is it?).

- $x_i = 1250 \text{ miles}$
- $v = -500 \frac{\text{mi}}{\text{hr}}$  (negative because its position is decreasing).
- So . . .  $x = 1250 \text{ mi} + (-500 \frac{\text{mi}}{\text{hr}}) \cdot t$  is a good model for the missile's position. Put in any value for time and you can calculate the position . . . for example, after a half-hour . . .  
 $x = 1250 \text{ mi} + (-500 \frac{\text{mi}}{\text{hr}})(0.5 \text{ hr}) = 1000 \text{ mi}$



Check your understanding once more by clicking on the check mark:

**My Notes for Constant Velocity Particle Model: The simplest model of motion in physics.**

Introduction:

One of the main focuses in physics is using models to describe how things move. To make keep things as simple as possible, in this unit, we will restrict ourselves to

- One \_\_\_\_\_, because . . .
- Objects whose motion will not be \_\_\_\_\_ or \_\_\_\_\_.

In this reading, we are going to develop and examine four representations of motion . . .

- Graphs of \_\_\_\_\_
- \_\_\_\_\_ of position during a motion.
- Graphs of \_\_\_\_\_
  - Symbolic representations ( \_\_\_\_\_ ) of motion.



How I rank my understanding after the first check quiz:

Clueless

Not sure

Got it!

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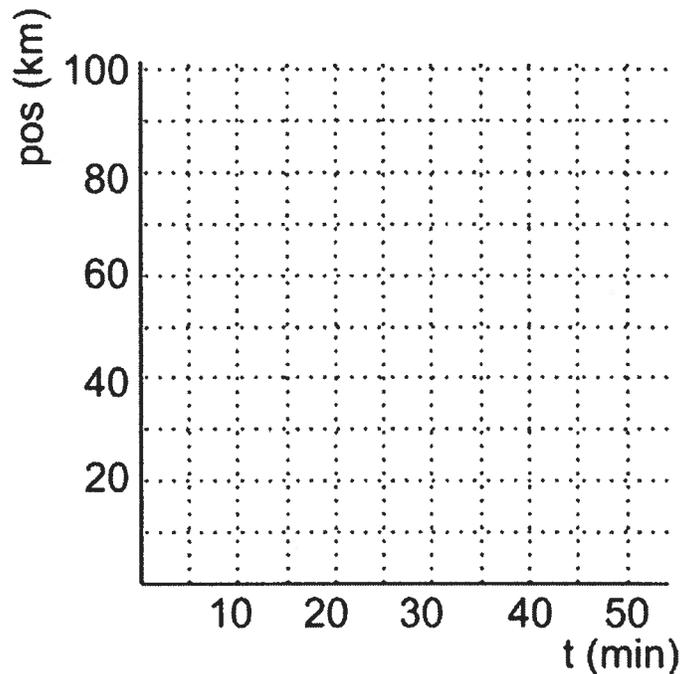
**Graphs of position and time:**

Imagine a fancy elevated high-speed commuter train moving between two fictional cities, Happyville and Busytropolis. Here's a possible experiment to create some data:

- The independent variable will be \_\_\_\_\_
- The dependent variable will be the \_\_\_\_\_

Fill in the data from the experiment presented on the website:

Time (minutes):					
Position (km):					

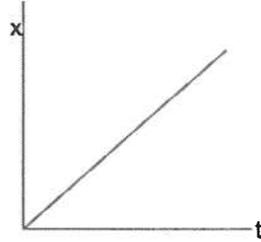


Now plot the data as a scatter plot:

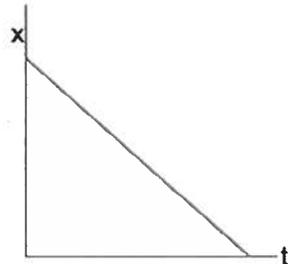
It's reasonable to think that a line is a good model for the motion of this train:

- What is the vertical intercept and what does it mean?
- What is the slope and what does it mean?
- If the train had been moving toward Happyville instead of away from it, how would the graph be different?
  - The starting position would be \_\_\_\_\_
  - The train's speed would be \_\_\_\_\_
  - The slope of the position vs. time graph would be \_\_\_\_\_
- How physicists think about this kind of graph. . .
  - The vertical axis tracks the \_\_\_\_\_, which is usually given the name \_\_\_\_\_ for one-dimensional motion.
    - The vertical intercept is the \_\_\_\_\_ and is often called \_\_\_\_\_ or \_\_\_\_\_.
    - How the object's position changes is called its \_\_\_\_\_. This usually designated by \_\_\_\_\_. ("Δ" is a symbol that means \_\_\_\_\_).
      - How is "distance" different from "displacement"?
  - The slope of a x vs. t graph is called the \_\_\_\_\_.

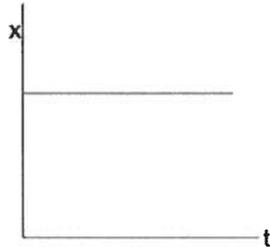
- A positive slope means the object is traveling \_\_\_\_\_.



- A negative slope means the object is traveling \_\_\_\_\_.



- A zero slope (horizontal line) means the object is \_\_\_\_\_.



- The \_\_\_\_\_ of the slope is the object's **speed**.
  - This is a graph of a straight line, which means that the object's velocity is not changing over time. We will call this model of motion the **C**\_\_\_\_\_ **V**\_\_\_\_\_ **P**\_\_\_\_\_ **M**\_\_\_\_\_.
- For CVPM motion, the shorthand version of the algebraic model is \_\_\_\_\_
  - The train in the first graph could be modeled as:
  - The train in the second graph could be modeled as:



How I rank my understanding after the first check quiz:

Clueless

Not sure

Got it!

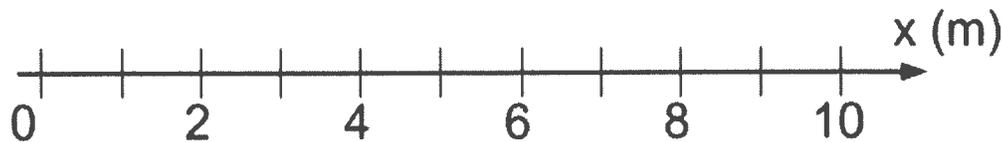
## **Motion Maps:**

Motion maps have the following features:

- There is some indication of the \_\_\_\_\_ and \_\_\_\_\_.
- Dots represent the \_\_\_\_\_ of the object at equal \_\_\_\_\_ intervals
- Arrows called \_\_\_\_\_ represent the \_\_\_\_\_ of the object at each second.
- If an object is moving forward, starting at 1 meter and moving at 3 m/s, draw the motion map:



- If the velocity is negative, the velocity vectors point \_\_\_\_\_.
- If the object is not moving, we show that time continues to flow by \_\_\_\_\_ with no \_\_\_\_\_.
- Objects that do several different kinds of constant velocity motion one after the other can be modeled, as well. For example, draw the motion map for an object that went forward from position zero at 4 m/s for 2 seconds, then stopped for 3 seconds, then went backwards at 1 m/s for 3 seconds, could be mapped like this:



How I rank my understanding after the first check quiz:

Clueless

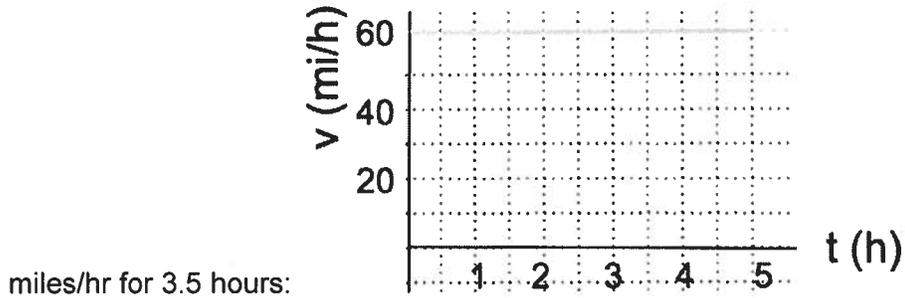
Not sure

Got it!

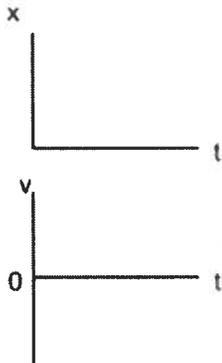
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**Graphs of velocity and time:**

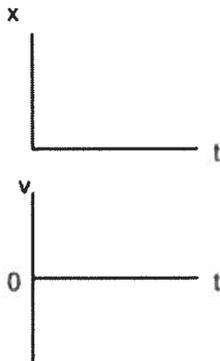
- Show on the graph how you could represent the displacement of a car moving at 60



- Let's explore how to translate  $x$  vs.  $t$  graphs to  $v$  vs.  $t$  graphs.
  - When you graph  $v$  vs.  $t$ , you are indicating the \_\_\_\_\_.
  - When the motion is constant velocity, the lines on a velocity vs. time graph are always \_\_\_\_\_ lines.
  - If the object is moving forward:



- If the object is moving backward:



- If the object is not moving, the  $v$  vs.  $t$  line is at \_\_\_\_\_:



How I rank my understanding after the first check quiz:

Clueless

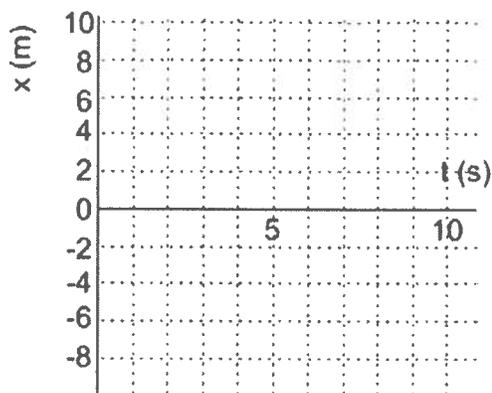
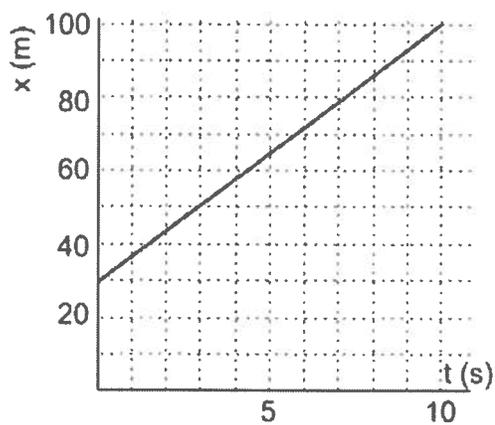
Not sure

Got it!

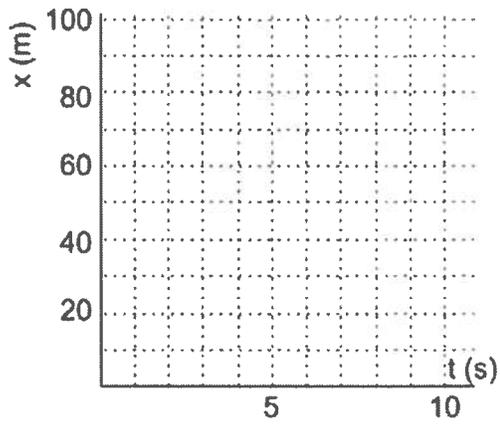
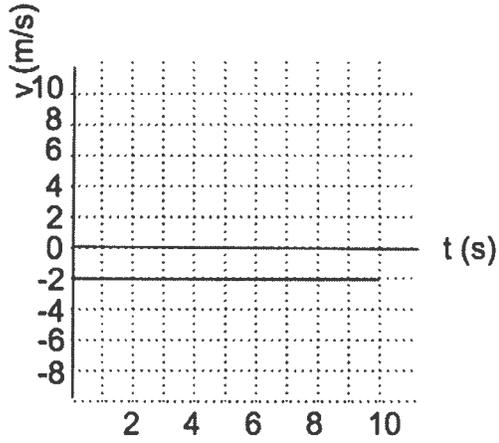
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To make the graphs quantitatively accurate, you will have to actually determine the slope. Let's try a couple of examples.

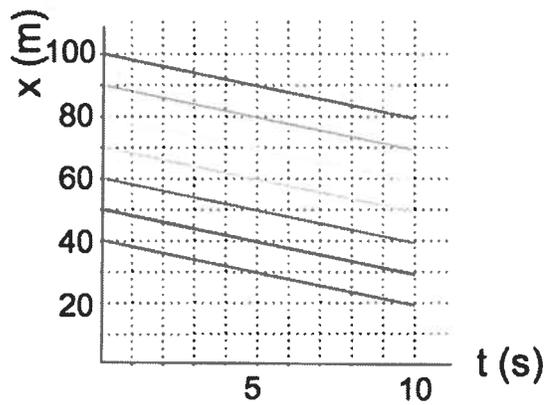
- Calculate the slope, then draw the velocity vs. time graph for this motion:



- Let's try one in reverse. Draw a position vs. time graph for this motion:



- Explain why each of these  $x$  vs.  $t$  graphs would have the same  $v$  vs.  $t$  graph





How I rank my understanding after the first check quiz:

Clueless

Not sure

Got it!

---

If a missile is moving toward its target at 500 mi/hr and its position when timing starts is 1250 miles from the target:

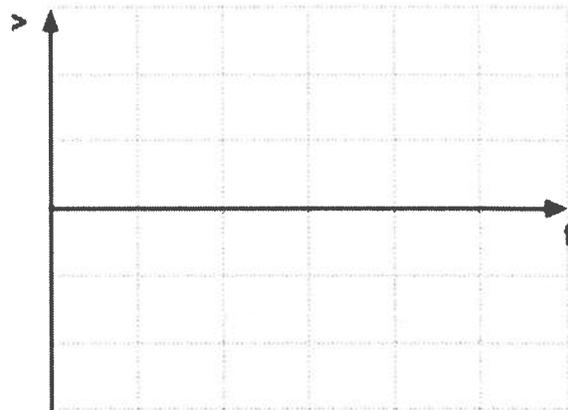
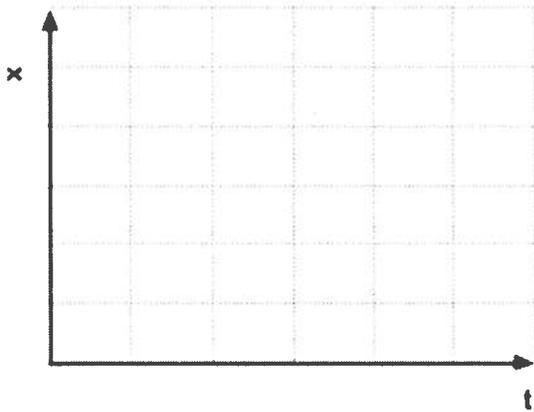
- Write the equation (mathematical or algebraic model) for this motion:
- Use this equation to find the position of the missile 0.5 hours after launch:

# CVPM Practice and Exploration #4



1. From the motion map above, answer the following:

- What can you conclude about the motion of the object?
- Draw a qualitative (not precise) graphical representation of  $x$  vs  $t$ .
- Draw a qualitative graphical representation of  $v$  vs  $t$ .



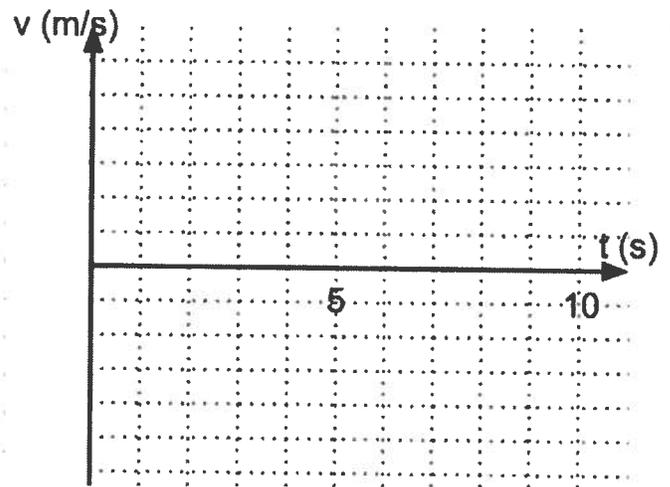
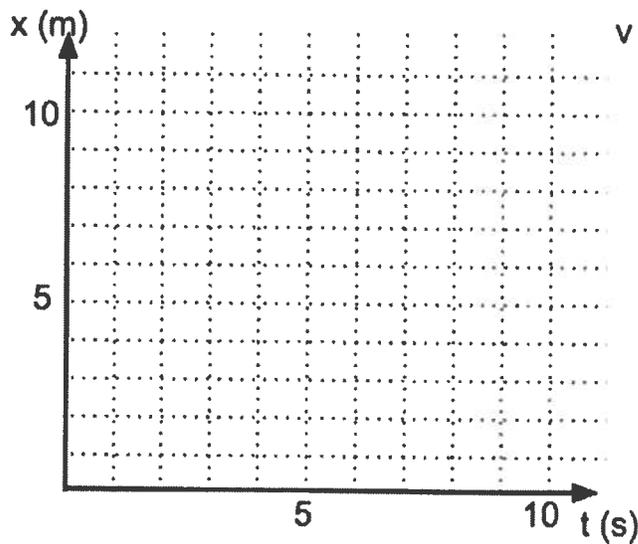
- Write a mathematical expression that represents the relationship between  $x$  and  $t$ , from your graph.
- Write a mathematical expression that represents the relationship between  $v$  and  $t$ , from your graph.
- Describe what the area under the curve in the  $v$  vs.  $t$  graph represents. Shade in this area.

2. From the position vs time data below, answer the following questions.

t (s)	x (m)
0	0
1	2
2	4
3	4
4	7
5	10
6	10
7	10
8	5
9	0

a. Construct a graph of position vs time.

b. Construct a graph of velocity vs time.



c. Draw a motion map for the object.

d. Determine the displacement from  $t = 3.0\text{s}$  to  $5.0\text{s}$  **using the v vs. t graph.**

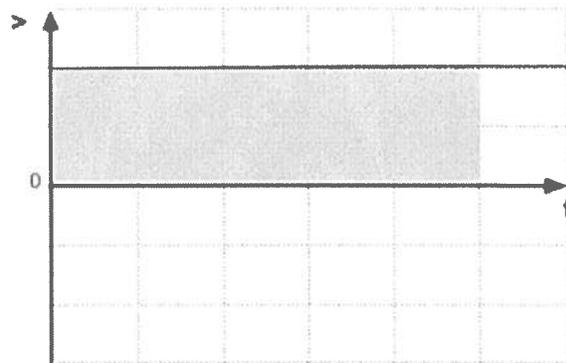
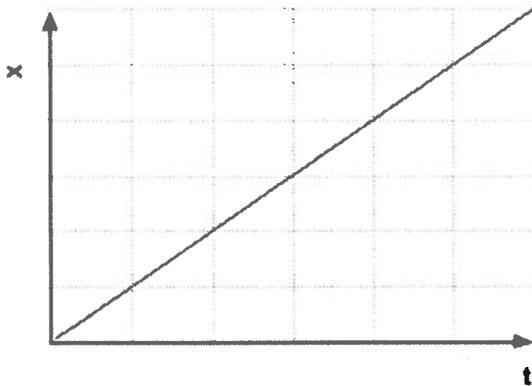
e. Determine the displacement from  $t = 7.0\text{ s}$  to  $9.0\text{ s}$  **using the v vs. t graph.**

# CVPM Practice and Exploration #4 Solutions



1. From the motion map above, answer the following:

- What can you conclude about the motion of the object? It's moving at a constant velocity, because every second it moves forward the same amount.
- Draw a qualitative (not precise) graphical representation of  $x$  vs  $t$ . It looks like the object started at position zero and is moving forward at a constant speed. So I'll put a straight line starting at zero with a positive slope on the  $x$  vs.  $t$  graph.
- Draw a qualitative graphical representation of  $v$  vs  $t$ . If the object has a constant forward velocity, its  $v$  vs.  $t$  graph should be a horizontal line above the 0 line.



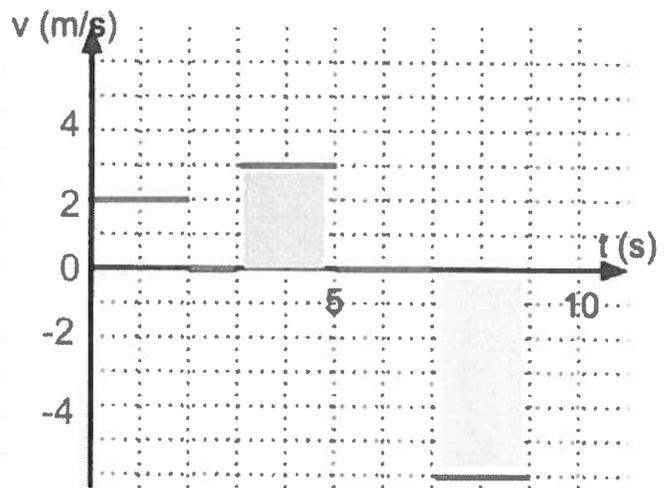
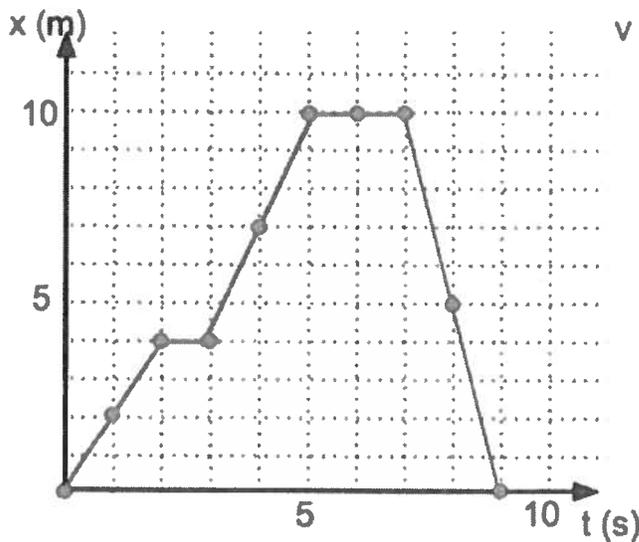
- Write a mathematical expression that represents the relationship between  $x$  and  $t$ , from your graph. It's a line. The vertical axis is  $x$ , the horizontal axis is  $t$ . the vertical intercept is zero and the slope of this graph is called velocity, so the equation is ...  $x = v \cdot t$
- Write a mathematical expression that represents the relationship between  $v$  and  $t$ , from your graph. It's a line. The vertical axis is  $v$ , the horizontal axis is  $t$ , the slope is zero and the vertical intercept is whatever the constant velocity is...  $v = k$
- Describe what the area under the curve in the  $v$  vs.  $t$  graph represents. The position of the velocity line tells how many meters the object moves forward every second (if velocity is meters per second). So every second that goes on, the object moves forward that many meters. So add all the seconds moved forward and you have the change in position. **Shade in this area.**

2. From the position vs time data below, answer the following questions.

t (s)	x (m)
0	0
1	2
2	4
3	4
4	7
5	10
6	10
7	10
8	5
9	0

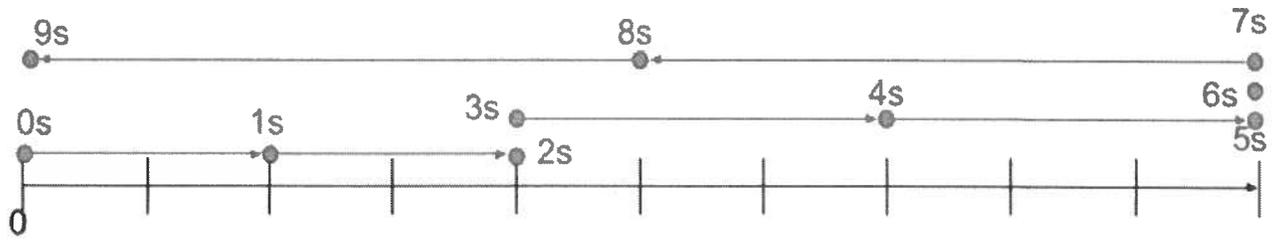
a. Construct a graph of position vs time. This is pretty easy. Just plot the points.

b. Construct a graph of velocity vs time. Velocity is the slope of the x vs. t. graph. For the first two seconds...  $\frac{4 \text{ m}}{2 \text{ s}} = 2 \frac{\text{m}}{\text{s}}$  For the third second, it's 0. For seconds 3 through 5...  $\frac{6 \text{ m}}{2 \text{ s}} = 3 \frac{\text{m}}{\text{s}}$ , then back to 0 from 5 through 7 seconds. For seconds 7 through 9...  $\frac{-10 \text{ m}}{2 \text{ s}} = -5 \frac{\text{m}}{\text{s}}$



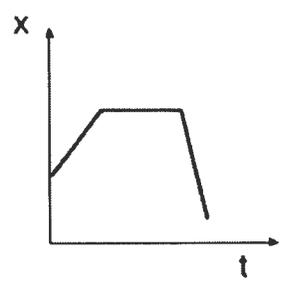
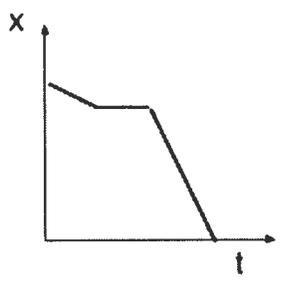
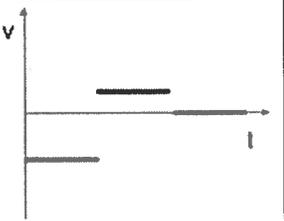
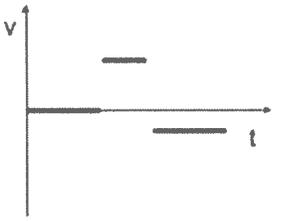
c. Draw a motion map for the object. The velocity vs time graph shows that the object jumps forward 2 m each second for two seconds, then stands still a second, then 3 m for two more seconds, then stops for two seconds, then jumps back 5 m each second for the last two

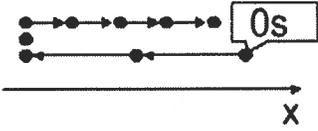
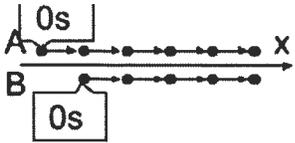
seconds.



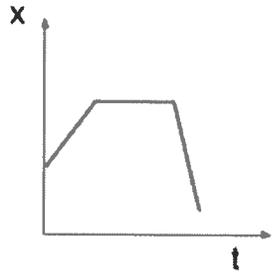
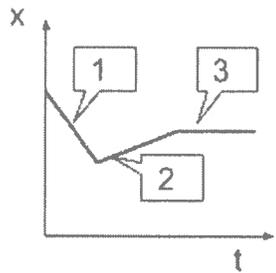
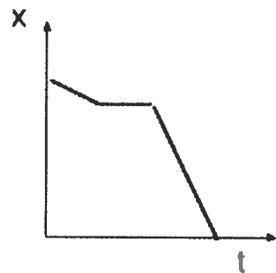
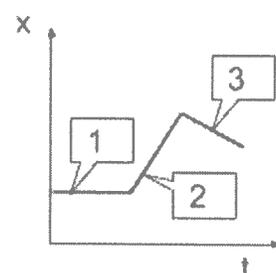
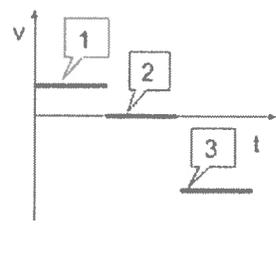
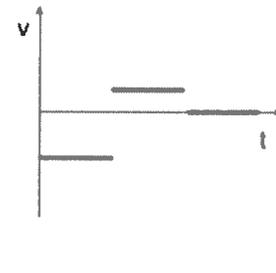
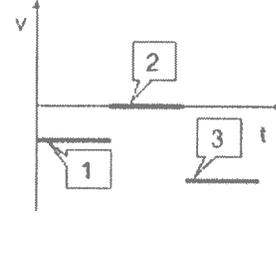
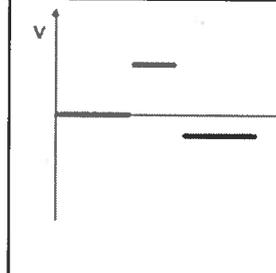
- d. Determine the displacement from  $t = 3.0\text{s}$  to  $5.0\text{s}$  **using the v vs. t graph**. The displacement is the area under the v vs. t graph. This is shaded pink above. The area is  $\left(3\frac{m}{s}\right)(2\text{ s}) = 6\text{ m}$
- e. Determine the displacement from  $t = 7.0\text{ s}$  to  $9.0\text{ s}$  **using the v vs. t graph**. The displacement is the area under the v vs. t graph. This is shaded green above. The area is negative because the velocity is negative.  $\left(-5\frac{m}{s}\right)(2\text{ s}) = -10\text{ m}$

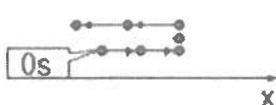
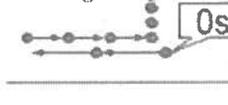
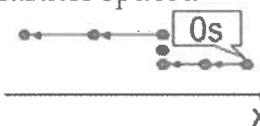
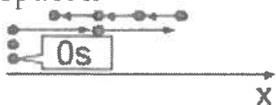
# CVPM Practice and Exploration #5

	1	2	3	4
x vs t				
v vs t				
words				
m. map				

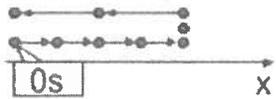
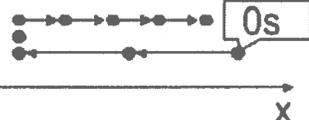
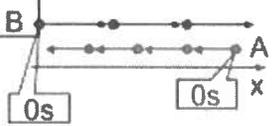
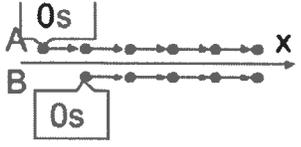
	5	6	7	8
x vs t				
v vs t				
words	Object moves with constant positive velocity for 4 seconds. Then, it stops for 2 seconds and returns to the initial position in 2 seconds.		Object A starts 10m to the right of the origin and moves to the left at 2 m/s. Object B starts at the origin and moves to the right at 3m/s.	
m. map				

# CVPM Practice and Exploration #5 Solutions

	1	2	3	4
<b>x vs t</b>		<p>(1) <math>v &lt; 0</math>, so slope <math>&lt; 0</math>; (2) <math>v &gt; 0</math> but less speed, so slope <math>&gt; 0</math>, but less steep; (3) <math>v = 0</math>, so slope <math>= 0</math></p> 		<p>(1) <math>v = 0</math>, so slope <math>= 0</math>; (2) <math>v &gt; 0</math>, so slope <math>&gt; 0</math>; (3) <math>v &lt; 0</math> and less speed, so slope <math>&lt; 0</math>, but less steep</p> 
<b>v vs t</b>	<p>(1) slope <math>&gt; 0</math>, so <math>v &gt; 0</math>; (2) slope <math>= 0</math>, so <math>v = 0</math>; (3) slope <math>&lt; 0</math> and steeper, so <math>v &lt; 0</math> and faster</p> 		<p>(1) slope <math>&lt; 0</math>, so <math>v &lt; 0</math>; (2) slope <math>= 0</math>, so <math>v = 0</math>; (3) slope <math>&lt; 0</math> and steeper, so <math>v &lt; 0</math> and faster</p> 	
<b>words</b>	The object moves forward at a constant speed, stops for a while, then moves backwards at a greater constant speed.	The object moves backward at a constant speed, then moves forward at a lower constant speed, then stops.	The object moves backward at a constant speed, stops for a while, then moves backwards again at a greater constant speed.	The object stands still for a while, then moves forward at a constant speed, then moves backwards at a lower constant speed.

<p>ni. map</p>	<p>(1) v vectors forward;(2) stack points during stop;(3) v vectors backward and spacing bigger than (1)</p> 	<p>(1) v vectors backward;(2) v vectors forward but closer spaced;(3) stacked points during stop</p> 	<p>(1) v vectors backward; (2) stacked points during stop;(3) v vectors backwards but farther spaced</p> 	<p>(1) stacked points while stopped;(2) v vectors forward; (3) v vectors backward but closer spaced</p> 
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	5	6	7	8
<b>x vs t</b>	<p>(1) <math>v &gt; 0</math>, so slope <math>&gt; 0</math>; (2) <math>v = 0</math> so slope <math>= 0</math>; (3) <math>v &lt; 0</math> and 2x as fast, so slope <math>&lt; 0</math> and 2x as steep</p>	<p>(1) <math>v &lt; 0</math>, so slope <math>&lt; 0</math>; (2) <math>v = 0</math>, so slope <math>= 0</math>; (3) <math>v &gt; 0</math> so slope <math>&gt; 0</math>, less steep than (1) b/c slower speed and doesn't return to starting position:</p>	<p>(A) <math>v &lt; 0</math>, so slope <math>&lt; 0</math>; (B) <math>v &gt; 0</math>, so slope <math>&gt; 0</math> and steeper b/c it's faster:</p>	<p>Both lines have the same slope b/c both objects have the same velocity. The line that represents B has a higher vert. intercept b/c B starts out ahead:</p>
<b>v vs t</b>	<p>This graph I can make by just plotting what's described. The trip back in (3) must be at twice the speed to cover the distance in half the time.</p>	<p>I did the description first and worked off that to show the direction by if <math>v &lt; 0</math> or <math>v &gt; 0</math> and speed by how far from the <math>v = 0</math> line:</p>	<p><math>v</math> vs. <math>t</math> graphs don't indicate position, so I only have to think about a <math>v &lt; 0</math> section for A and a <math>v &gt; 0</math> section for B. The <math>v &gt; 0</math> section s/b farther from the <math>v = 0</math> line, b/c it's moving faster:</p>	<p>Since <math>v</math> vs. <math>t</math> graphs don't show position, and both are moving at the same velocity, the lines should both be in the <math>v &gt; 0</math> region and the same distance from <math>v = 0</math>:</p>

words	Object moves with constant positive velocity for 4 seconds. Then, it stops for 2 seconds and returns to the initial position in 2 seconds.	From the motion map, the object has a large negative $v$ for 2s, then stops for 2s, then has a lower forward $v$ for 4s and doesn't get all the way back to the start	Object A starts 10m to the right of the origin and moves to the left at 2 m/s. Object B starts at the origin and moves to the right at 3m/s.	From the motion map, both objects are moving with the same velocity, because their directions are the same and the jumps between seconds are the same. B just has a head start.
m. map	<p>forward <math>v</math> vectors for 4s (4 dots); 2 stacked dots (2s); 2 dots go all the way back to the start</p> 		<p>I start A to the right, moving left with jumps that represent 2m each, I start B at left with jumps that represent 3m each:</p> 	

# Optional VPython and Glowscript Project

Welcome to the computational side of physics!

I'm including in this week some optional work for you if you have access to a computer and internet. It is a really great way to reinforce what you know about CVPM, while at the same time learning about how to write code (or practice it, if you already code). The language you will be using is a version of Python, which is very relevant, since it is the language in which Instagram, Google, Netflix, Spotify, Uber and many other world-class platforms were written.

If you don't have the capability, you simply will not be able to do these projects. Don't worry about it...that's just life right now.

Programming is fun and rewarding, but can also be frustrating, because programming languages are so picky about details. (They have to be--that's just how programming works.) Don't be surprised or give up if you get confused or if you write some code you think should be working but won't. Almost always, there's some little detail, like a command that's spelled wrong or a missing semicolon, that is the problem. I can also help you work through these problems during office hours.

As you go through this project, I definitely recommend that you access the documents through the Moodle so you can make use of the embedded links easily. Also, when you finish the final project, you'll have to go to the Moodle to learn how to screencast it and submit it.

# VPython and Glowscript

Welcome to the computational side of physics!

## Lesson 2: Loops, Physics, and Graphs

*Expected time: 1-2 hours*

**Objectives: In this tutorial you will learn**

- How to copy previous work into a new program
- How to write and execute a 'while' loop
- How to use loops to update the position of an object, thus to make it move
- How to create and manipulate graphs

### Table of Contents

Lesson 2: Loops, Physics, and Graphs

Building from your previous work

The Basic Loop

Real Physics with loops

Add a Graph (or two)

Final Product

### Building from your previous work

Lesson 2 builds on what you learned in previous lessons. In fact, everything builds on previous work. You will learn about using a while loop, modifying and updating the position attribute, and designing the limits of your system to show what they are supposed to show.

When you are using your previous work as a starting point for new work, it is important to both **preserve the original work** and **create the new work**. Sometimes you even want to save a version of a program and continue work on a new version so that you have a reference and can go back to a version that you know worked (or at least failed in a predictable way).

In your programs list (recall you can get there by clicking on your username), you should first hit the copy button for your Hello World program so it is easily accessible in a new tab, then name the copy Constant Velocity. Copy and modify the header code (the information we added in lesson 1, such as program name, authors, etc) from the Hello World program into the Constant Velocity program. *Note: This highlights the importance of comments. If you copy old code into a new program, comments will help you know why any particular line of code exists, which will help later in what you have to do to modify it.*

# The Basic Loop

VPython/Glowscript uses the **while** loop as the basic loop function.

The purpose of a loop is to execute specific operations over and over again, either indefinitely or while a certain condition is true. For example, if your job is to stack wood in piles of 100 logs, you would go through a loop -- pick up a log, put it on the pile, add one to your mental counter, see if you hit 100, and if not then then pick up a new log.

In Glowscript/VPython-type code, this would look like this:

```
logs = 0
while logs < 100:
    pickuplog
    placelog in pile
    logs = logs + 1
```

Of course, that's **not real python code** -- but the structure is similar and you can read through it and see how it works.

1. The `logs` variable is a counter that starts at zero.
2. The `while` statement establishes the conditions for the loop. So long as the number of logs is less than 100, do another loop.
3. The `pickuplog` and `placelog` commands (not real commands!) represent your actions of picking up a log and placing it in a pile.
4. The `logs = logs + 1` line increments the counter.

So, what would a real Glowscript/VPython loop look like? Type the following code into Glowscript. I've added notes to help see what the loop does.

```
1 GlowScript 2.1 VPython
2 x=0
3 while x<21:
4     rate(5)
5     x=x+1
6     print("x=",x)
```

*Some things to note:*

- Line 2: `x=0` defines zero as the starting `x` value.
- Line 3: "while" tells glowscript to loop, and `x<21` tells the program to run the loop only if `x` is less than 21. The colon is required.
- Line 4: `rate` defines the frequency of the loop; 5 means a maximum of 5 loops every second. Also note that everything in the loop is indented. This is the indicator to Glowscript that those are parameters of the loop.
- Line 5: `x=x+1` indicates that every time the loop runs, take the value of `x`, add one, and assign that as the new value of `x`.
- Line 6: the "x=" in quotes is actually text, whereas the `x` after the comma, without quotes, is the variable defined previously.

After running the program once, try changing the values of 21, 5, and 1 to see what happens. Then, with the rate set to 10, unindent the print command. This changes from printing each loop to printing only after all the loops have completed. Does it work as you expected?

When you are done you can delete everything but line 1 of your code; that was just to see how loops work. Now we'll create a loop to make an object move.

## Real Physics with loops

We'll start with two objects; a box and the 'ground,' which is really just for reference and in Glowscript is really just another box.

Name and define the floor as the following;

```
floor=box(pos=vec(0,-.2,0), size=vec(10,.02,.4))
```

Run your code to see what it looks like. The purpose of the y position of -.2 is to allow the 'car' to run at y=0 above the 'ground.'

Next name a box 'car', make each side 0.2 in length (units in VPython are arbitrary but consistent), and give it a color. Its default position is (0,0,0), so you don't have to give it a pos definition. [Refer to Lesson 1](#) as needed for these steps.

Now that we have created 'car,' we can define some attributes of it. Below shows code to set the initial position (again, not actually needed, but this way we can change it later if we want) and initial velocity of the car (though it won't move just because we define a velocity; we have to update its position as we'll see in a bit). We also defined the initial time as zero and the change in time, dt, as 0.01s.

```
car.v=vec(.5,0,0)
car.pos=vec(0,0,0)
t=0
dt=0.01
```

Next the loop. Recall that slope of a position vs. time graph is  $\frac{\Delta x}{\Delta t}$  and it is equal to the velocity if acceleration is zero (and the average velocity over  $\Delta t$  if acceleration is not constant). From  $\frac{\Delta x}{\Delta t} = v$  we can rearrange to get  $\Delta x = v\Delta t$ , which then becomes  $x_f = x_i + v\Delta t$ . For example, if a car moves for 3 seconds at 5 m/s when it started at the 2 meter mark, It will end at the 17 meter mark. The problem is that we can't put that equation directly into Glowscript/VPython, so we have to translate it. We also use the fact that Glowscript/VPython will allow you to update a variable to a new value with an equation that uses that same variable, such as  $x = x + 1$ . So, our CVPM position equation would become something like this:

```
car.pos=car.pos+car.v*dt
```

**\*\*\*NOTE\*\*\*** *The above equation is going to be super, super important as we continue with VPython and physics.*

Then we must 'add a log' again so our counter increases, as we want to know how much time has elapsed.

```
t=t+dt
```

Add these equations into a 'while' loop that runs when t is less than 20 seconds and with a rate 300 (otherwise it's terribly slow). Don't forget the colon at the end of the while statement.

Before you run the program, let's think about what happens here.

As long as  $t$  is less than 20, the loop will continue to cycle. Every cycle it takes the current position (the `car.pos` on the right side of the equation), adds the velocity times the small change in time (so in this case,  $0.5 * 0.01 = .005$ ), then sets that as the new position, then takes the current time ( $t$  on the right side of the  $t = t + dt$  equation), adds the small change in time of 0.01, and sets it as the new time. Then it repeats. Take that new position, add  $v * dt$ , set as new new position, find the new time, repeat. Thus our program updates the position of the car and the time elapsed every 0.01 seconds, which is what makes it move.

So now run your program; your 'car' should move to the right across the screen. What happens when it reaches the end of the floor? It looks like the floor shrinks. Really the camera is zooming out to keep the car in its view. We'll look at a couple of different ways to approach that later.

Next we'll add something cool; a trail to the car so we can 'see' that it's moving at a constant velocity. Add the following into the car definition, in the parentheses, near the top of your code;

```
make_trail=True, trail_type='spheres', trail_radius=.05, interval=40
```

Now run your program; you should see something that looks like a motion map as the car travels. Play around with your code a bit; try changing the attributes of the trail, the rate, velocity, initial position, or ending time of the while statement. Investigate the relationship between  $dt$  and the rate of the loop. Figure some stuff out, and have fun!

Lastly, add the following near the top of your code;

```
scene.autoscale = False
```

This is one way to address objects going off the screen; prevent the scene from autoscaling to see the object. The negative is that objects are simply not viewable once they get off the screen, unless you zoom out manually with two finger scroll on your (Mac) trackpad. You can also change it back to autoscaling by changing to True.

## Add a Graph (or two)

Our next step is to add a graph to match the car's motion. First we have to set the attributes of the graph window. Near the start of your code add the following;

```
c1=gcurve()
```

This creates a graph, but *does not* create any points on that graph. To do that we need to tell the loop to add a point to the graph every time it cycles, so in your loop add

```
c1.plot(t, car.pos.x)
```

This adds the point  $(t, x)$ , where  $x$  is the  $x$  position of the car (note: VPython knows that `pos.x` is the  $x$  position of the car, pretty cool, right?). It does this on *every loop*. So really your graph is made up of a bunch of points with time separations of 0.01s (or whatever your  $dt$  is). The `gcurve` function, however, connects these dots, whereas `gdots` does not.

Run your program, and you should see the graph changing as the program runs; remember, it is adding a point every iteration of the loop. Change `gcurve` to `gdots` and change the rate to less than 5 to really see how the graph is being created.

Change your rate back to 300, then add graph g1 and change your c1 definition to that below;

```
g1 = graph(xmax=20, xmin=0, ymax=10, ymin=-10, xtitle='time', ytitle='x position')
c1=gcurve(graph=g1)
```

Run your program to see how this changes the graph itself.

It's important to understand how the above works; now we are first creating a graph called g1, then creating a curve c1 on that graph (gcurve plots on graph g1). To see how this is working, create a second curve c2, added to g1 as well, and add a new plot to your loop so it plots y position vs. time as well as x position vs time. You should see both curves (the new one horizontal at zero) when you run your program.

Next create a new graph, g2, with the same attributes as g1, but change the ytitle to 'y position'. Then change your code so c1 graphs on g1 and c2 graphs on g2, and make sure to have both c1 and c2 plots in your loop.

## Final Product

Now that you have graphs figured out we are going to finalize your program. The end result is going include your constant velocity car, as well as;

- A graph of both x and y position
- A graph of both x and y velocity
- The x and y positions will be color coded, as will the velocities
- A general title on the graph to note the color code

You will need four gcurves now; c1 and c2 on g1 for x and y position, and c3 and c4 on g2 for x and y velocities. Modify c1 and c2, create c3 and c4, and retile g1 and g2 for that purpose. Note that to plot x velocity you use car.v.x.

Next give c1 and c3 a color by adding color=color\_\_\_\_\_ inside the gcurve parentheses, so that the x and y curves on the same graph are different colors. Then inside both g1 and g2, add title='\_\_\_\_\_' where the blank is filled with a description of which curve is which color. Use the physics definitions, not the code, for these descriptions; for example, 'x position is yellow and...'

Once you have both graphs, try changing the initial position and velocity of the car to see how the graphs change, including negative values.

Last of all, go back and add some comments highlighting the various aspects of your code.

# VPython and Glowscript

Welcome to the computational side of physics!

## Lesson 2b: Applying computational physics to CVPM scenarios

*Expected time: 1-2 hours*

**Objectives:** In this tutorial you will

- Create programs to simulate 6 motion scenarios
- Make a screencast of your simulations as they run

### Table of Contents

Lesson 2b: Applying computational physics to CVPM scenarios

What's the objective?

Setting the stage

What motions am I simulating?

Final Product

### What's the objective?

Lesson 2b builds on what you learned in the previous two lessons and gives you an opportunity to demonstrate your knowledge of CVPM by simulating the motions of two objects. These are the same motions we first used in the CVPM unit as we began to explore multiple graphical representations of CVPM. This will be a lot more open-middle than the first two lessons, so you will have to figure out a lot of stuff on your own, based on what you learned there and what you know about CVPM.

## Setting the stage

You may wish to copy the program you made in lesson 2 to save time. The basic structure will be similar, but you will have to add, subtract and modify some lines for the new project.

---

All the motion simulations you will be doing will involve positions that will be between 0 and 10 units. For this reason, we will make the “floor” conform to that size, so start all six simulations with this line of code:

```
floor=box(pos=vec(5,-.2,0), size=vec(10,.02,.4))
```

This establishes a floor that is...

- 10 units long in the x axis... (the y and z values don't exactly have to be what I chose)
  - with its center at 5 units, so that...
  - the left edge is aligned with  $x = 0$
  - a little below the x axis (where  $y = 0$ ), so that the motion on the x axis is visible
- 

You will need to define two objects for these simulations, so the next two lines of code should each establish...

- a box that is 0.2 units on a side
  - as the box moves it drops a trail of spherical points of radius 0.05 units
  - trail points should be dropped every one time unit (to simulate 1 per second, like a motion map. You will be using a loop with a dt value of 0.01, so you'll have to figure out how to set your “interval” value.)
  - one line should establish the red box and one should establish the blue box.
- 

To keep the motions clear so you can see what both are doing at the same time, when you establish the starting position of the blue object, always make the y value a little higher than 0. That way your trails won't be right on top of each other. (I chose  $y = 0.5$  and it worked pretty nicely.)

---

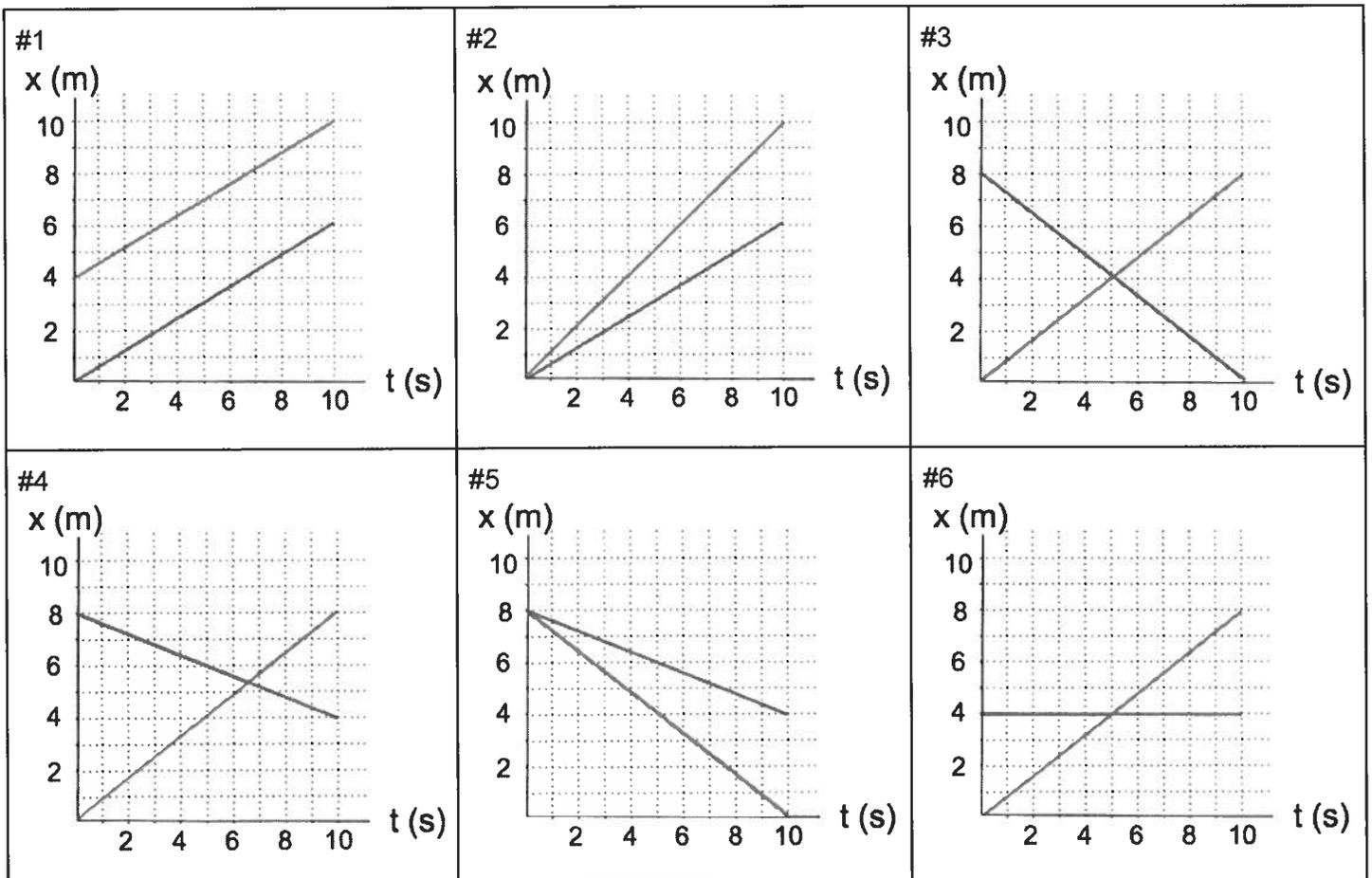
When you run your loops, run them to simulate only the motion shown in the graphs you are given. All the graphs will range from 0 - 10 seconds.

---

If you copied your program from lesson 2, you can take out the code for making graphs. We are only interested in the motion we see when we run the program, not the graph.

## What motions am I simulating?

Your programs should simulate the following 6 motion scenarios. (I'd recommend getting the first one running well, then make copies and modify for the other 5.)

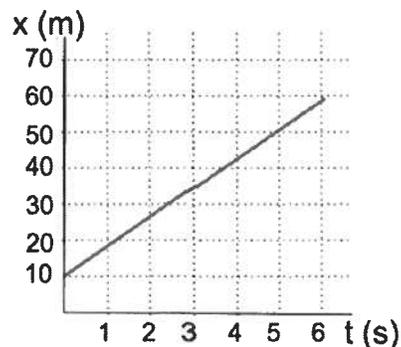


## Final Product

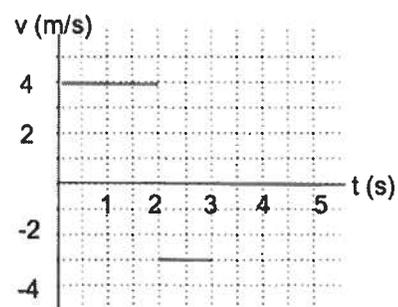
Once you have your code written, create a screen recording of the code and then you running the program for each set of motions. Make sure that your code is on the screen for at least 2 seconds, so I have a chance to pause it and read what you've written. Submit the screen recording as directed.

## UNIT II: Review

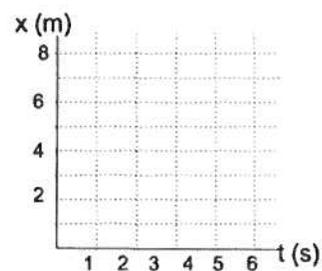
1. Consider the position vs time graph at right.
  - a. Determine the average velocity of the object.
  - b. Write a mathematical equation to describe the motion of the object.



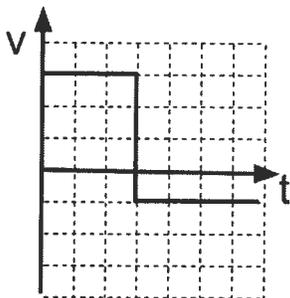
2. Shown at right is a velocity vs time graph for an object.
  - a. Describe the motion of the object.
  - b. Draw the corresponding position vs time graph. Number the  $x$ -axis.
  - c. How far did the object travel in the interval  $t = 1\text{s}$  to  $t = 2\text{s}$ ?



- d. What is the total displacement? Explain how you got the answer.



3. Johnny drives to Wisconsin (1920 miles) in 32 hours. He returns home by the same route in the same amount of time.
  - a. Determine his average speed.
  - b. Determine his average velocity.
  - c. Compare these two values and explain any differences.
4. Consider the  $v$  vs  $t$  graph below.



- a. Describe the behavior of the object depicted in the graph.
- b. Draw a motion map that represents the behavior of the object.



5. A race car travels at a speed of 95 m/s. How far does it travel in 12.5 s? Use the appropriate mathematical expression and show how units cancel. (Keep the proper number of sf's.)

## UNIT II: Review

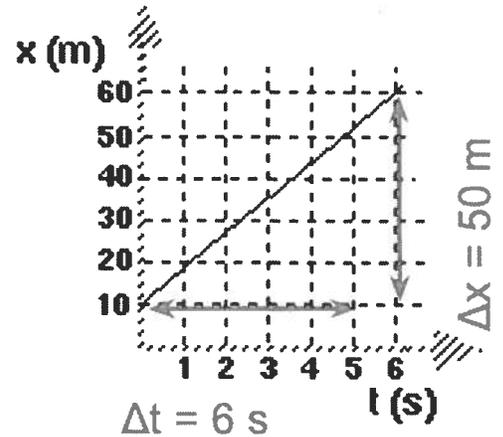
1. Consider the position vs time graph at right.

a. Determine the average velocity of the object.

The slope of the x vs. t graph is the velocity, so:  $v = \frac{\Delta x}{\Delta t} = \frac{50 \text{ m}}{6 \text{ s}} = 8.3 \text{ m/s}$

b. Write a mathematical equation to describe the motion of the object.

$$x = (8.3 \text{ m/s})t + 10 \text{ m}$$



2. Shown at right is a velocity vs time graph for an object.

a. Describe the motion of the object.

The object moves forward for 2 seconds at a constant speed of 4 m/s, then moves backwards for 1 second at a constant speed of 3 m/s, then stands still for 2 seconds.

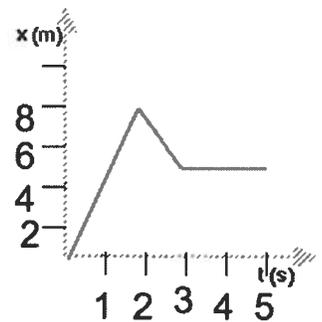
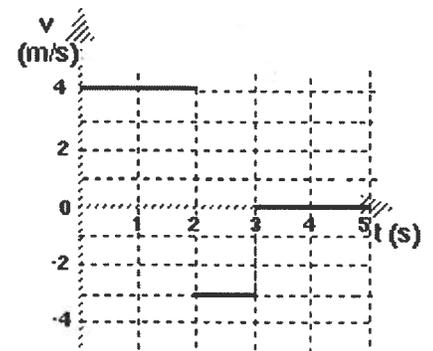
b. Draw the corresponding position vs time graph. Number the x-axis. My graph assumes that the object started at x = 0.

c. How far did the object travel in the interval t = 1s to t = 2s?

At t = 1s its position was 4 m and at t = 2s, its position was 8m, so it travelled 4 m.

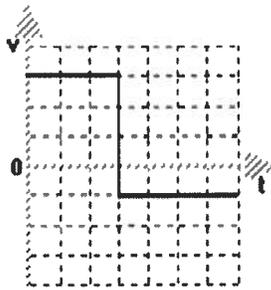
d. What is the total displacement? Explain how you got the answer.

At t = 0s, its position was 0m and at t = 5s, its position was 5m, so the total displacement was 5m.

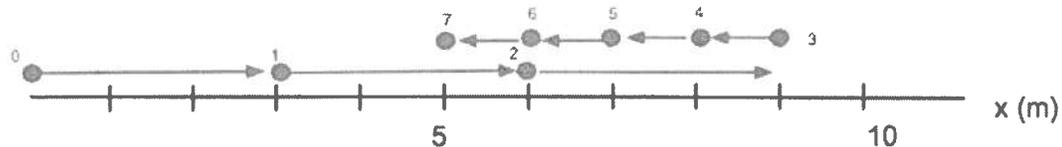


3. Johnny drives to Wisconsin (1920 miles) in 32 hours. He returns home by the same route in the same amount of time.
- Determine his average speed.** Speed doesn't depend on direction (sign, in vector terms), only on distance (magnitude, in vector terms). So we could calculate speed as  $v$  by  $v = \frac{\sum|\Delta x|}{\Delta t}$ . In this case, this is  $v = \frac{1920 \text{ miles} + 1920 \text{ miles}}{32 \text{ hours} + 32 \text{ hours}} = \frac{3840 \text{ miles}}{64 \text{ hours}} = 60 \text{ mi/hr}$ .
  - Determine his average velocity.** Velocity does depend on direction and is defined as  $v = \frac{\Delta x}{\Delta t}$ . In this case, velocity is  $v = \frac{1920 \text{ mi} + (-1920 \text{ mi})}{32 \text{ hr}} = \frac{0 \text{ mi}}{32 \text{ hr}} = 0 \text{ mi/hr}$ .
  - Compare these two values and explain any differences.** The average velocity is zero because the two movements cancel each other out to result in 0 overall displacement.

4. Consider the  $v$  vs  $t$  graph below.



- Describe the behavior of the object depicted in the graph.**  
Assuming the velocity unit is m/s and the time unit is s, it moves forward at 3 m/s for 3 seconds, then turns around and moves backwards at 1 m/s for the next 4 seconds.
- Draw a motion map that represents the behavior of the object.**



5. A race car travels at a speed of 95 m/s. How far does it travel in 12.5 s? Use the appropriate mathematical expression and show how units cancel. (Keep the proper number of sf's.) Since  $v = \frac{\Delta x}{\Delta t}$ , it must also be true that  $\Delta x = v\Delta t = \left(\frac{95 \text{ m}}{1 \text{ s}}\right) \left(\frac{12.5 \text{ s}}{1}\right) = 1200 \text{ m}$ . We would keep 2 sig. figs. in our answer, because there are only 2 sig. figs. in 95 m/s.

## Constant Acceleration Particle Model: A more advanced model of motion in physics.

As you are reading this . . .

- *If you have the capability to access this document on the class Moodle page, your experience will be better. If you can't, you can still get a lot out of it, but obviously you will not be able to access the check quizzes and other links.*
- *you will see some words that are bold and in italics. These are words that will be important to look up if you don't understand them. Some are links to dictionary definitions, but if that doesn't clear it up for you, do some exploring until you think you get what the word means.*
- *you will see "check for understanding" links. I've put them there to help you see if you're picking up what was in the section you've just finished. This reading is really of almost no value if you just keep reading through it without understanding, so use these little quizzes to check your understanding. If you find that you are mostly answering the questions incorrectly, it would probably be a good idea to go back, re-read and take the check. Repeat if necessary.*

### Introduction:

One of the main focuses in physics is using models to describe how things move. This reading activity is a first look at how physics models a slightly more advanced type of motion than that where the object moves at the same velocity all the time. In this case, we are going to allow objects to speed up and slow down. We are still going to restrict ourselves to one dimension. This means there is only forward and back. No right and left. No up and down.

In this reading, we are going to re-visit the same four representations of motion we used for the Constant Velocity Particle Model of motion (CVPM), except in a different order this time.

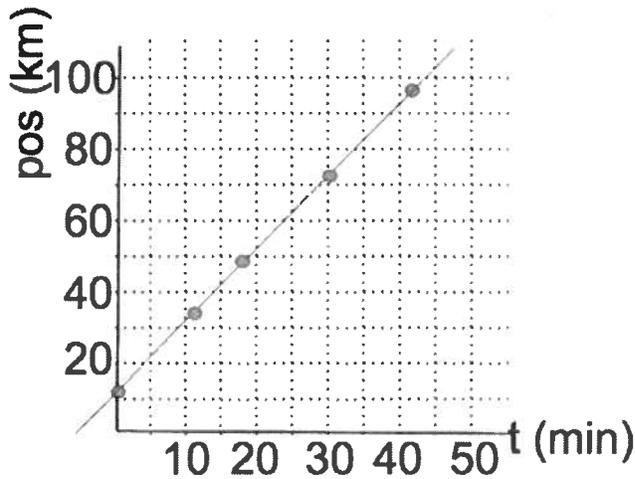
- Maps of position during a motion.
- Graphs of velocity and time. ( $v$  vs.  $t$ )
- Graphs of position and time. ( $x$  vs.  $t$ )
- Symbolic representations (equations) of motion.



Check your understanding now by clicking on the check mark:

In the reading for Constant Velocity motion, we started the model-building with an elevated high-speed commuter train moving between two fictional cities, Happyville and Busytropolis. We were taking a long-range view of this motion and we used data to determine that the train

was moving at constant velocity. This was modeled by this graph of position vs. time.

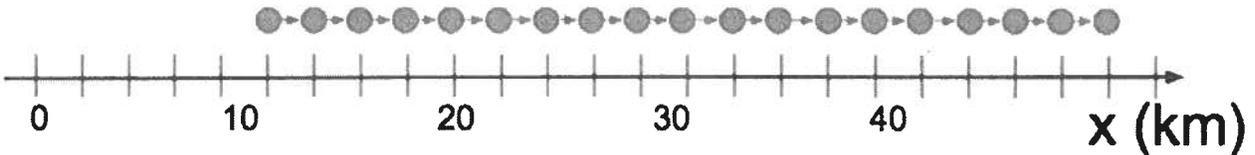


We used the slope to define the velocity, which was 2 km/min. When we built that model, the clock started when the train was already 12 km from Happyville and was already moving at a constant speed. But if the train stopped in Happyville to pick up passengers, its velocity was zero. By the time it had reached the 12 km marker, it was moving at a positive velocity . . . in other words, its velocity must have changed between those two points. That's the part of the motion that we are interested in now.

**Motion Maps:**

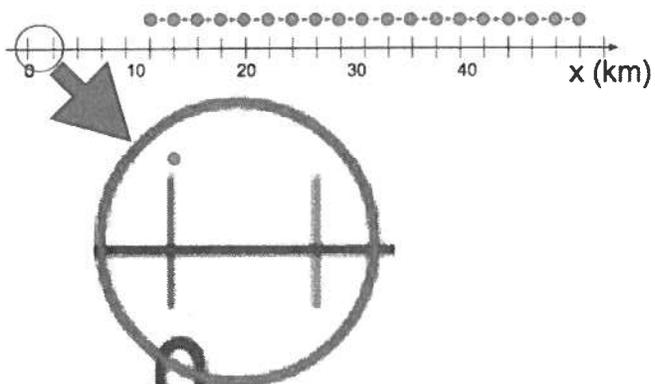
If we drew a motion map for that train, moving at a constant velocity of 2 km/min, we would include these features:

- If each dot represented a minute, we would see dots equally spaced 2 km apart.
- The velocity vectors would point in the forward direction and be equally sized . . .

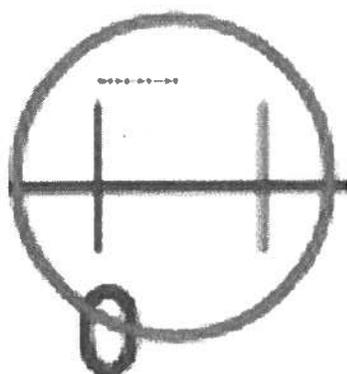


But if we had started tracking position when the train was sitting in the station, the map would have to include a dot with no velocity vector.

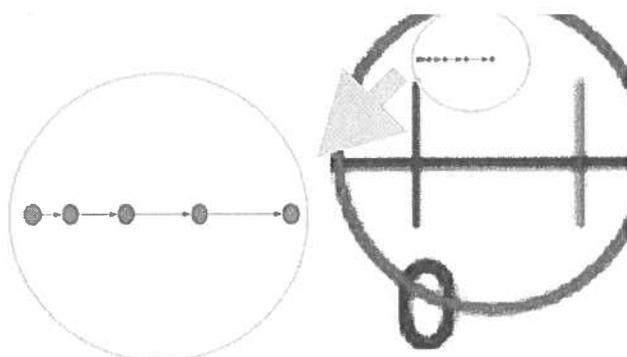
Let's say it takes one kilometer for this train to get up to its cruising speed and it covers this first kilometer in four minutes. Let's zoom in on the first kilometer only:



If this change in velocity happens in a steady way, the motion map for that first km might look like this



Here's a closer look . . .



Check your understanding now by clicking on the check mark:



## Position vs. time graphs

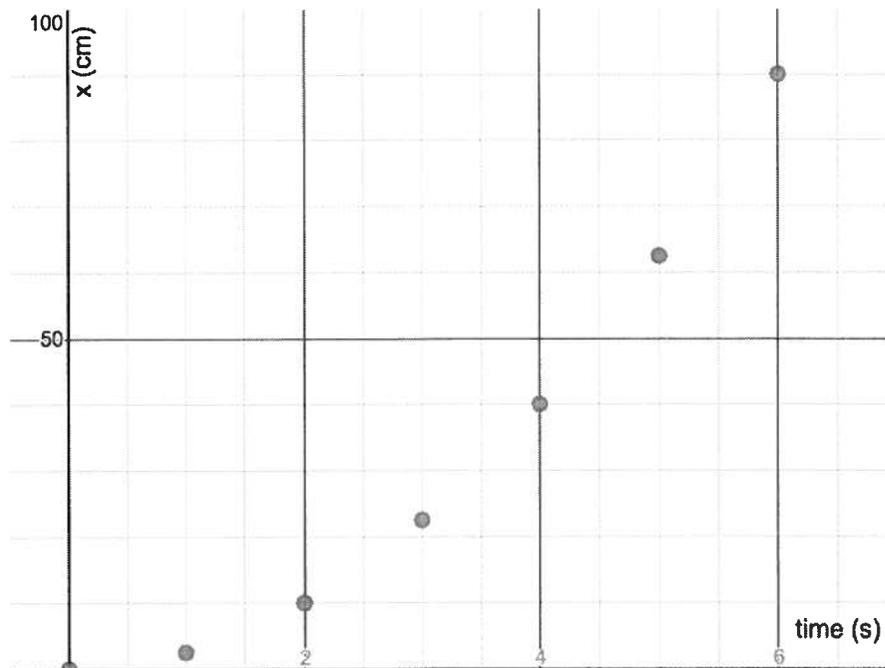
When we look at this section of the motion map, it is clear that the position jumps for each second are not the same. If you think about how that would translate to a position vs. time graph, you might imagine that graph would not be a straight line.

Let's look at another motion situation like this in greater detail.

Imagine that a small toy car rolls down a shallow ramp and its position is measured every second for the first six seconds of its motion. This is the result:

time (s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
x (cm)	0.0	2.5	10.0	22.5	40.0	62.5	90.0

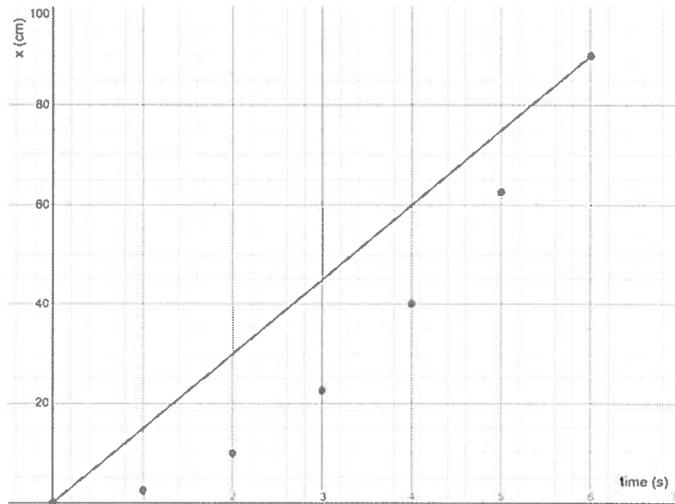
Let's plot this data on a position vs. time graph . . .



It's clearly not a straight line, as you might have thought. From studying model-building, you might have a hunch about what to do to linearize this data and produce a good mathematical model for this motion. Before we go there, however, let's think about what we know about this motion already from studying CVPM.

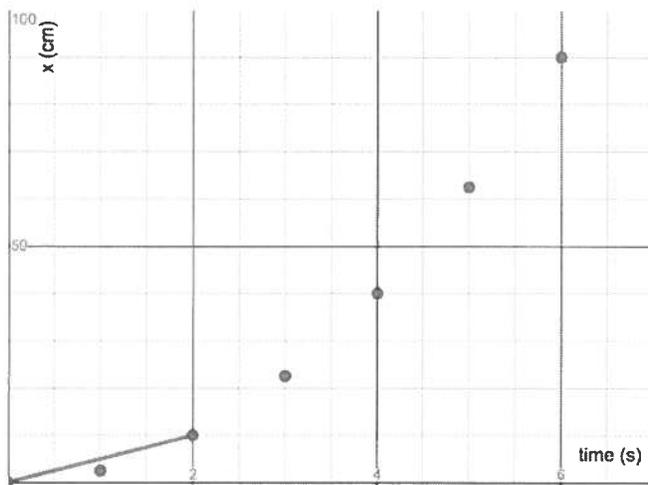
We know that the average velocity of an object is represented graphically by the slope of the  $x$  vs.  $t$  graph. This is constantly changing here, but let's try to uncover some stuff about the nature of this change.

First, let's determine the average velocity over the whole 6 seconds:



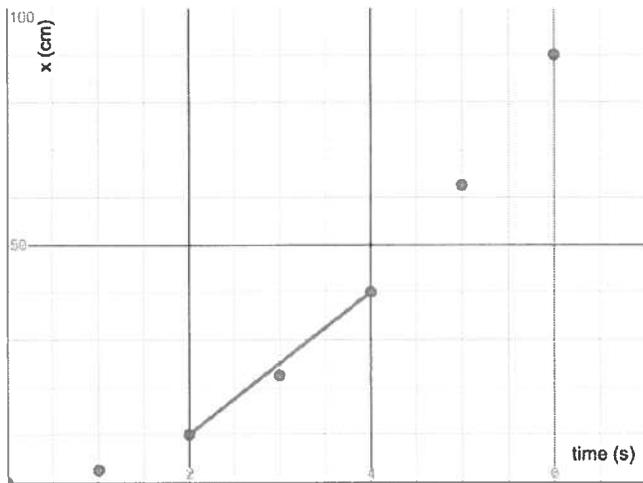
$$v = \frac{\Delta x}{\Delta t} = \frac{90 \text{ cm}}{6 \text{ s}} = 15 \frac{\text{cm}}{\text{s}}$$

Now, let's find the velocity during the first 2 seconds:



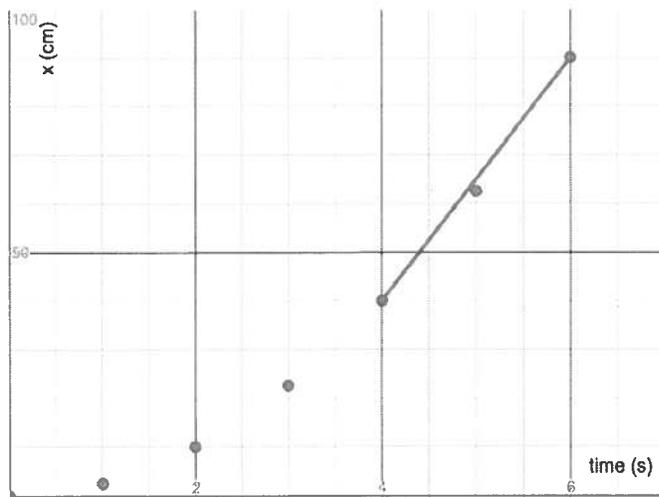
$$v = \frac{\Delta x}{\Delta t} = \frac{10 \text{ cm}}{2 \text{ s}} = 5 \frac{\text{cm}}{\text{s}}$$

Now the next two seconds:



$$v = \frac{\Delta x}{\Delta t} = \frac{30 \text{ cm}}{2 \text{ s}} = 15 \frac{\text{cm}}{\text{s}}$$

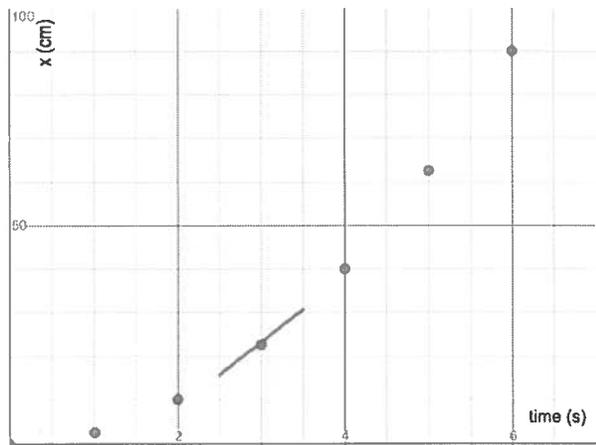
Now the last two seconds:



$$v = \frac{\Delta x}{\Delta t} = \frac{50 \text{ cm}}{2 \text{ s}} = 25 \frac{\text{cm}}{\text{s}}$$

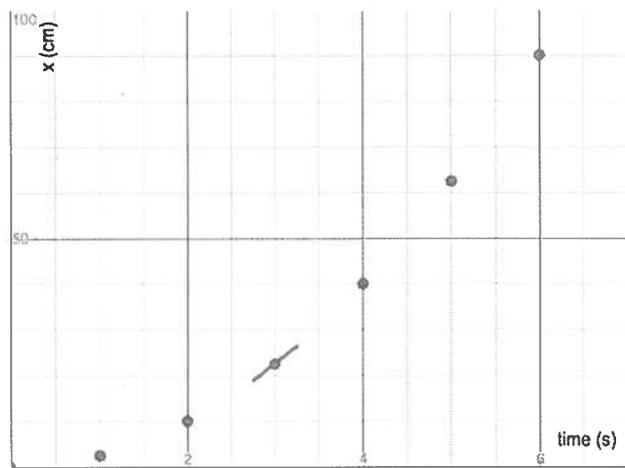
The velocity keeps increasing, as you'd expect, because the car is speeding up. If you were paying attention, you probably noticed that the average velocity in the middle two seconds was the same as the overall average velocity.

Let's try something else--making the time interval only one second overall, but still centered on the middle (3rd) second...



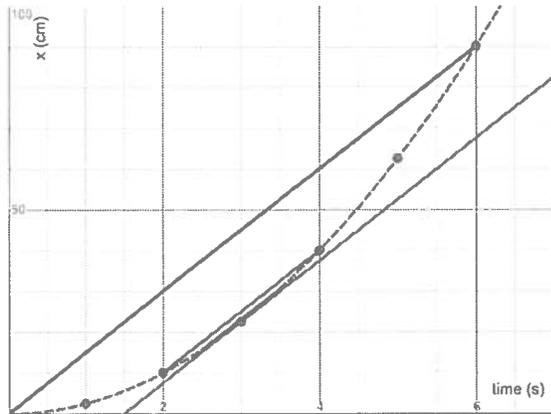
$$v = \frac{\Delta x}{\Delta t} = \frac{15 \text{ cm}}{1 \text{ s}} = 15 \frac{\text{cm}}{\text{s}}$$

The velocity over this interval was also the same. Now let's shrink it to a half-second:



$$v = \frac{\Delta x}{\Delta t} = \frac{7.5 \text{ cm}}{0.5 \text{ s}} = 15 \frac{\text{cm}}{\text{s}}$$

These velocities are the same because all these lines are parallel. In fact, no matter what time interval we chose, as long as it was centered on the same second, it would always be parallel. If we shrunk the time interval down so that it was only one single point, the line we'd have is called the tangent line, which is shown here in green, along with the other parallel lines, and it would have the same slope as the others.



The velocity found by the slope of that tangent line is called the "instantaneous velocity", which means the velocity at that instant.

An important lesson from this model that we will use soon is this:

***"When an object is changing its velocity at a constant rate, its instantaneous velocity is the same as the average velocity when it is exactly half-way through the change."***



Check your understanding now by clicking on the check mark:

### **Velocity vs. time graphs**

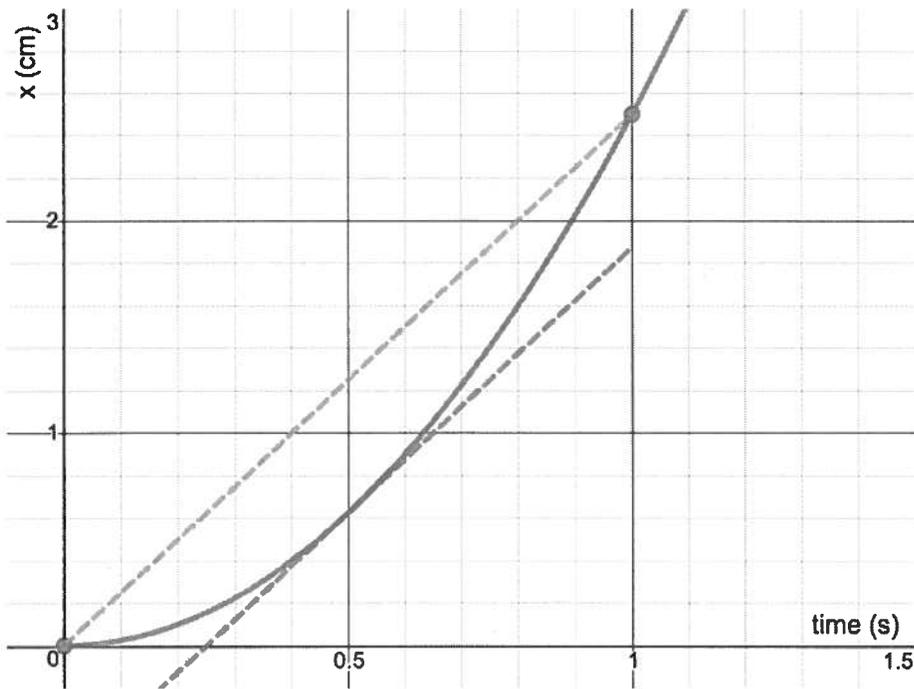
Since the slopes of the tangent lines for our toy car are constantly increasing as time increases, the instantaneous velocity of the car is constantly increasing.

Now let's take a closer look at precisely how the velocity is changing by looking at each second as a separate time interval . . .

Returning to our data for the toy car, let's zero in on the interval from 0.0 s to 1.0 s . . .

Time (s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
x (cm)	0.0	2.5	10.0	22.5	40.0	62.5	90.0

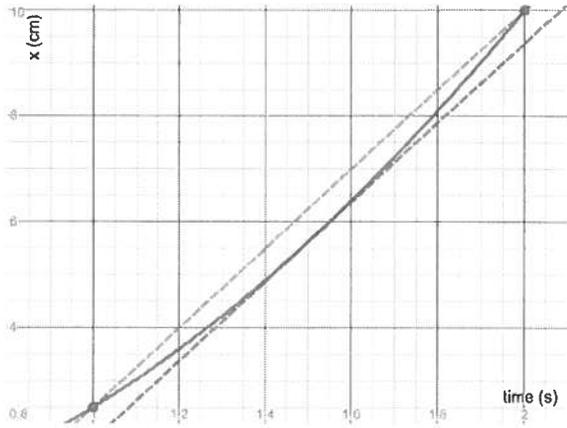
Let's zoom in on the position vs. time graph for just this second . . .



$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{2.5 \text{ cm}}{1.0 \text{ s}} = 2.5 \frac{\text{cm}}{\text{s}}$$

Since the car moved forward 2.5 cm during this second, the average velocity during that second is 2.5 m/s.

But we have also learned that the instantaneous velocity half-way through the motion (which is at 0.5 s) is the same as the average velocity, so the velocity at the instant of 0.5 s is 2.5 m/s. Now let's move to the next second . . .



$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{10 \text{ cm} - 2.5 \text{ cm}}{2.0 \text{ s} - 1.0 \text{ s}} = \frac{7.5 \text{ cm}}{1.0 \text{ s}} = 7.5 \frac{\text{cm}}{\text{s}}$$

Again, since the instantaneous velocity halfway through this second (at 1.5 s) is the same as the average velocity over this second, the velocity at 1.5 s is 7.5 cm/s.

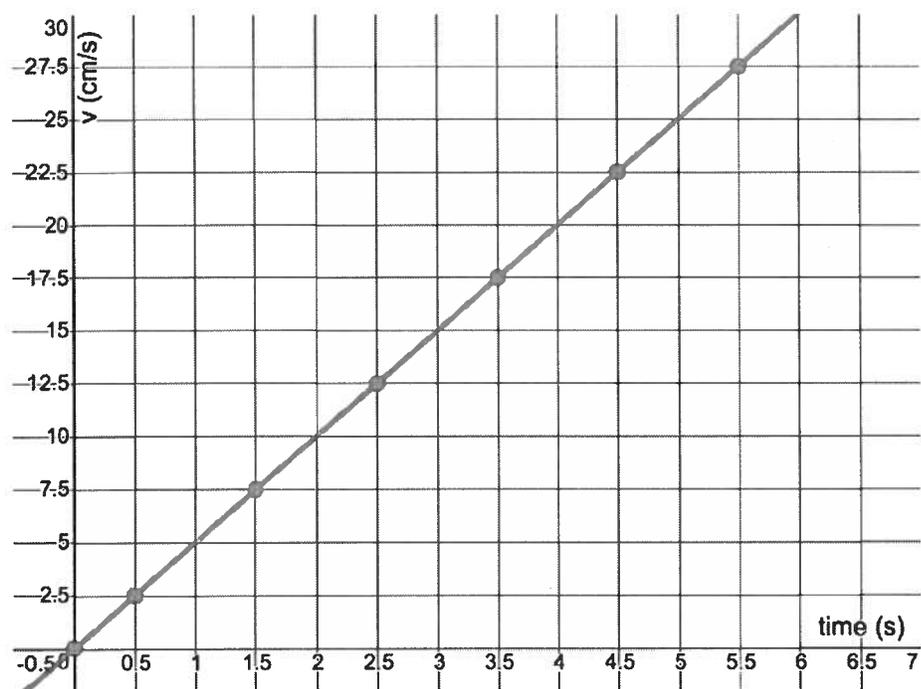
We can continue doing this through the remaining seconds of the motion. The results are in this table.

t (s)	x (cm)	$\Delta x$ (cm)	$\Delta t$ (s)	$v_{av}$ (cm/s)
0.0	0.0	↕ 2.5	↕ 1.0	↕ 2.5
1.0	2.5	↕ _____	↕ _____	↕ _____
2.0	10.0	↕ 7.5	↕ 1.0	↕ 7.5
3.0	22.5	↕ _____	↕ _____	↕ _____
4.0	40.0	↕ 17.5	↕ 1.0	↕ 17.5
5.0	62.5	↕ _____	↕ _____	↕ _____
6.0	90.0	↕ 22.5	↕ 1.0	↕ 22.5
		↕ 27.5	↕ 1.0	↕ 27.5

Since the velocities at the midpoints of each of these seconds of motion are the same as the average velocities during those seconds, we can make a table of velocities and times from the table above.

time (s)	0	0.5	1.5	2.5	3.5	4.5	5.5
v (cm/s)	0	2.5	7.5	12.5	17.5	22.5	27.5

When we plot this data as a scatter plot and do a linear regression, we find that a line is a very good model for this relationship:



$v_1 \sim mt_1 + b$	
STATISTICS	RESIDUALS
$r^2 = 1$	$e_1$ plot
$r = 1$	
PARAMETERS	
$m = 5$	$b = 0$

$$v = \left( 5 \frac{\text{cm}}{\text{s}} \right) t \quad \text{or} \quad v = \left( 5 \frac{\text{cm}}{\text{s}^2} \right) t$$

The slope of this line has a special name, which is a word you have probably heard before. We call the slope of any velocity vs. time graph "acceleration". In symbols . . .

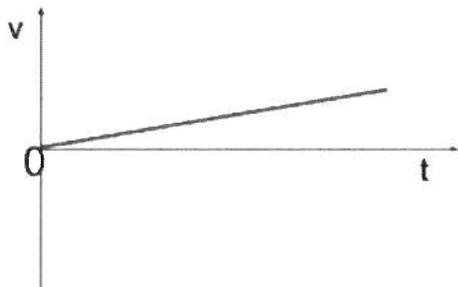
$$a \equiv \frac{\Delta v}{\Delta t}$$

Since this is a line, the equation should take the form of any line, with the slope being "a" for acceleration and the vertical intercept being "v<sub>o</sub>", for starting velocity:

$$v = v_o + a\Delta t$$

The kind of motion we are studying in this reading is limited to situations like this, where it is possible to model velocity vs. time with a straight line. When we view objects as particles and restrict ourselves to this type of motion, that is called the Constant Acceleration Particle Model of motion, or CAPM, for short.

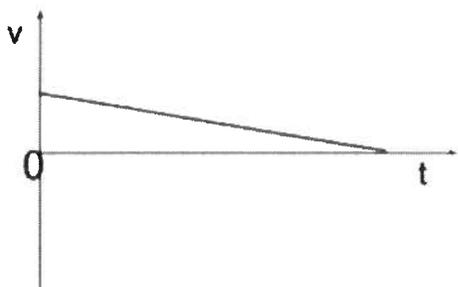
Now let's compare some v vs. t graphs and see what we can tell about their motions from the appearance of the graphs. The basic shape of the graph we just made looks like this:



This represented an object that was moving forward and speeding up. When we studied CVPM, we recognized that when an object was moving in the forward direction, the velocity is above the zero velocity line, which this one does.

However, back then, our graphs always were horizontal lines, because velocity was always constant. Here that's not the case. What happens as time goes on is that the value of velocity gets farther from the zero velocity line, which means it is speeding up.

Let's look at another object's motion on a v vs. t graph . . .

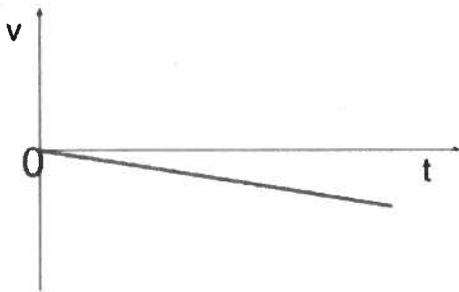


If your first impression is that this object is moving backwards, you're making a very common beginner's mistake. After studying CVPM, many people instantly look at the downward slope of the graph and associate it with the shape of a position vs time graph for an object moving backwards at constant speed.

But this is not a position vs. time graph; instead it's velocity vs. time. Let's dig into what this means...

At time = 0, the value of the velocity is positive, which means the object is moving forward. As time goes on, the velocity gets closer to the zero line, which means the object is slowing down. Eventually the velocity reaches 0, which means the object has come to a stop.

Let's look at another one. Before the explanation, think about what it means that the line is below the  $v = 0$  line. Also think about what it means that the line is getting farther from the  $v = 0$  line. Can you describe the motion correctly?



This object's velocity starts at zero, which means it is not moving. As time goes on, the velocity gets farther from the  $v = 0$  line, which means it is speeding up. But it is always below the  $v = 0$  line, which means velocity is negative, so the object is going backwards.



Check your understanding now by clicking on the check mark:

Now it's time to put the graphical models of  $x$  vs.  $t$  and  $v$  vs.  $t$  together and learn how to translate between them.

A very powerful tool for doing this has the nickname of "pencil test". It's actually a crude way to do a calculus technique called differentiation. But don't be scared by the word calculus--this is pretty easy to do.

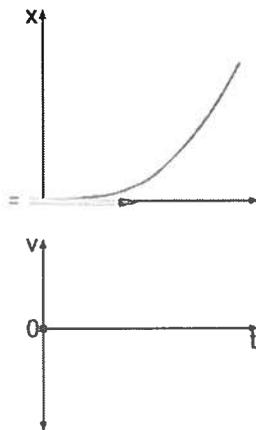
The idea is to hold (or imagine holding) a pencil at a tangent to the  $x$  vs.  $t$  graph as you increase the value of time and looking at two things:

- What direction is the pencil sloping? (This tells you if the object is moving backwards or forwards at that instant.)
- How is the steepness of the pencil's slope changing? (This tells you if the object is speeding up or slowing down.)

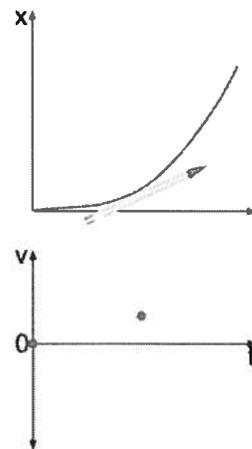
Positive slope means positive velocity.

Bigger steepness means farther from the  $v = 0$  line.

Let's try a couple of examples. Here, the pencil starts with a 0 slope, so velocity is zero.



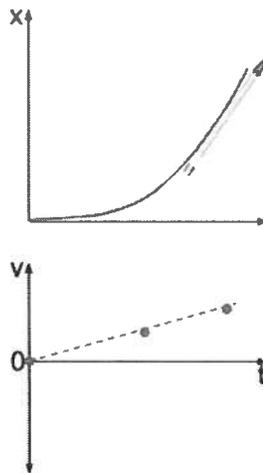
As time increases the slope becomes positive, so the velocity is in the



positive region of the graph (above the  $v = 0$  line).

As time increases even

more the slope becomes even steeper, so the object is speeding up, which means the velocity

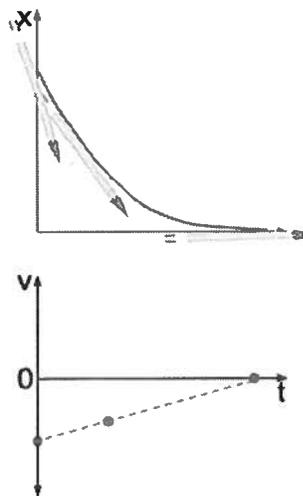


line gets farther away from the  $v = 0$  line.

In the next example, the pencil starts with a negative slope, so velocity is below the  $v = 0$  line.

As time increases the slope becomes less steep, so the object is slowing down, which means the velocity line gets closer to the  $v = 0$  line.

Until the very end, the pencil is always sloping negatively, so the velocity line continues to be below the  $v = 0$  line.



Check your understanding now by clicking on the check mark:

## Adding acceleration to our graphical models

### Part I: a vs. t graphs:

Acceleration is the rate of change of velocity over time, so acceleration could be plotted against time on a new kind of graph.

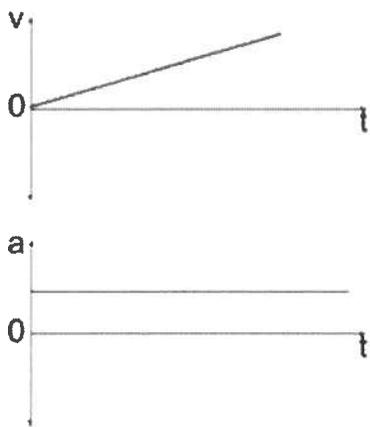
The slope of velocity vs. time is the acceleration.

Fortunately, the slopes of velocity vs. time graphs for CAPM are always straight lines, so that means that the accelerations are never changing.

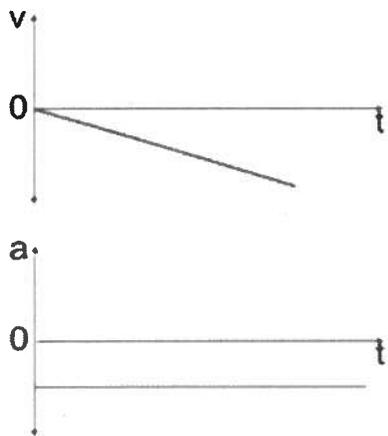
So the lines on an a vs. t graph will always be horizontal.

We just have to decide if the horizontal line will be above, below or on the  $a = 0$  line. This is not hard. A positive slope on a v vs. t graph means a positive acceleration. A negative slope on a v vs. t graph means a negative acceleration. A slope of zero on a v vs. t graph means acceleration is 0.

For example, the v vs. t graph below shows a positively sloping straight line, so the a vs. t graph shows a constant, positive value for acceleration:



In this next  $v$  vs.  $t$  graph we see a negatively sloping straight line, so the  $a$  vs.  $t$  graph shows a constant, negative value for acceleration:

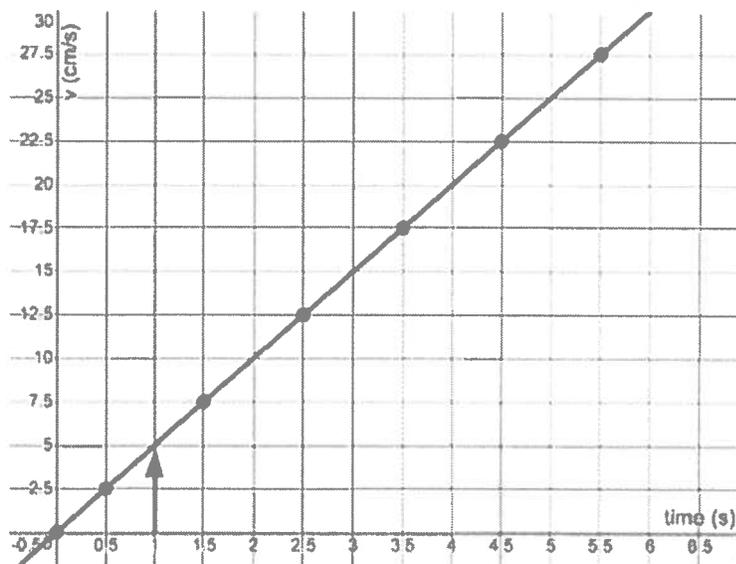


Check your understanding now by clicking on the check mark:

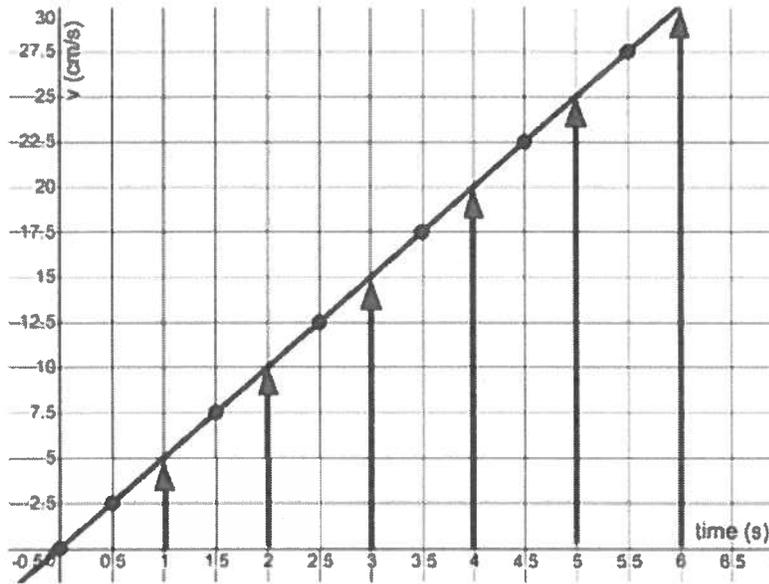
## Part II: Enhancing Motion Maps

Let's go back to the  $v$  vs.  $t$  graph for the toy car.

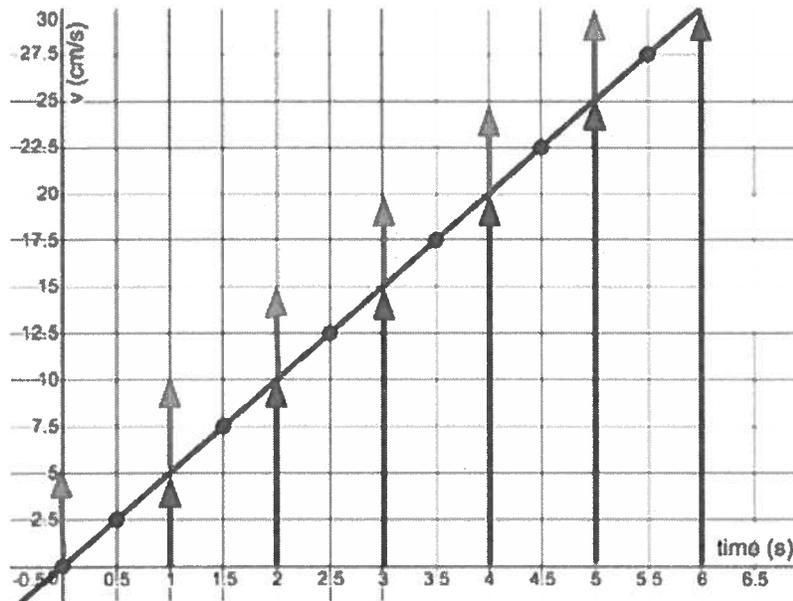
If I draw a line from the  $v = 0$  line up to the velocity line at the first second, the length of that line is the space on the velocity graph between zero and the velocity the object actually has at that second.



If I do that for each second, the lengths get longer and the increase in length looks like it isn't random.



The change in the length of the velocity vector each second is literally the acceleration. This change turns out to be constant and is shown by the red vectors shown below:



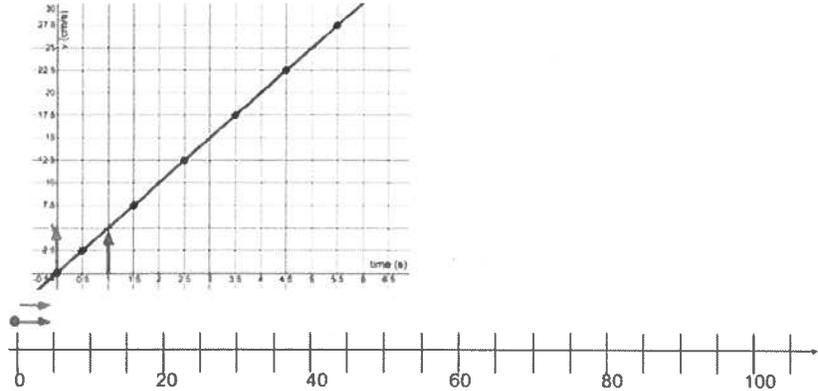
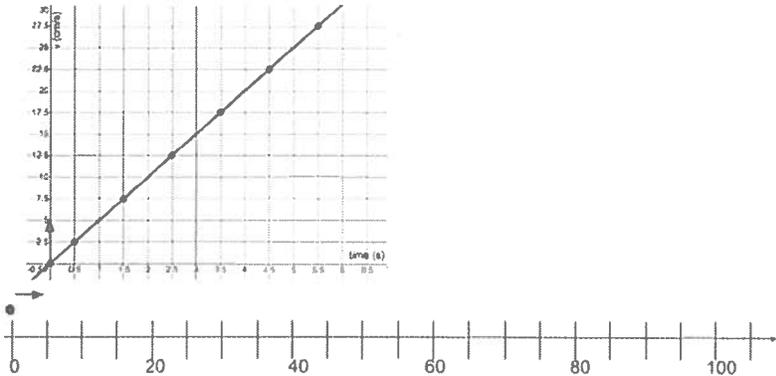
If the object is speeding up, like we see here, the acceleration vectors are in the same direction as the velocity vectors. In other words, acceleration changes the speed by making it larger.

Now let's enhance our motion map by showing the acceleration vectors.

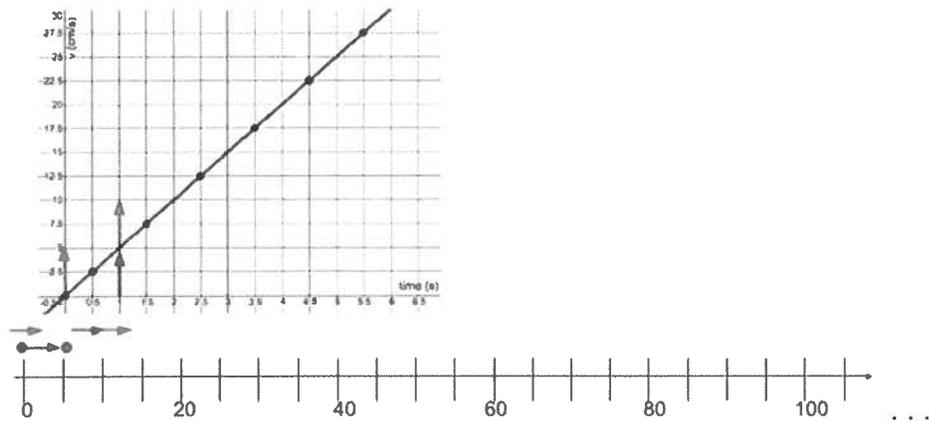
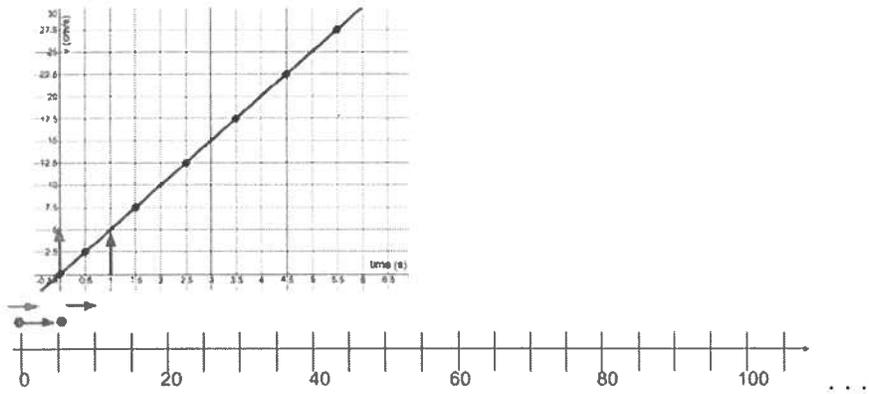
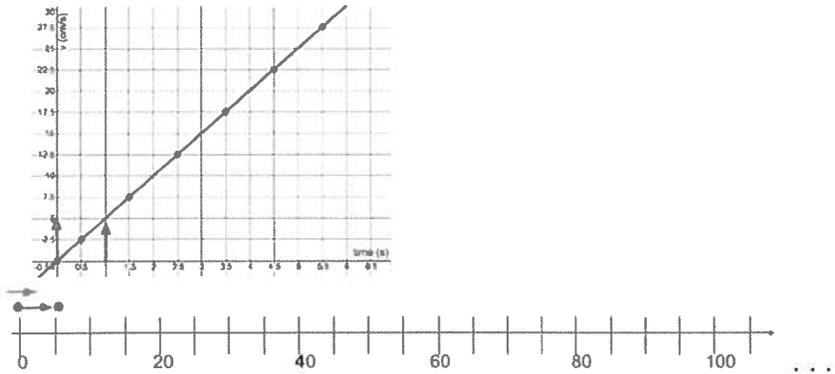
The blue vectors point in the positive direction (which on our map is to the right). They grow in length with each second by the length of the acceleration vector. Let's track this for a few seconds:

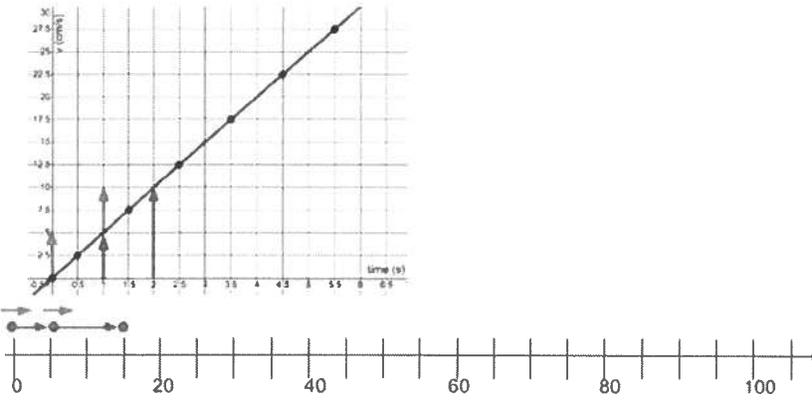
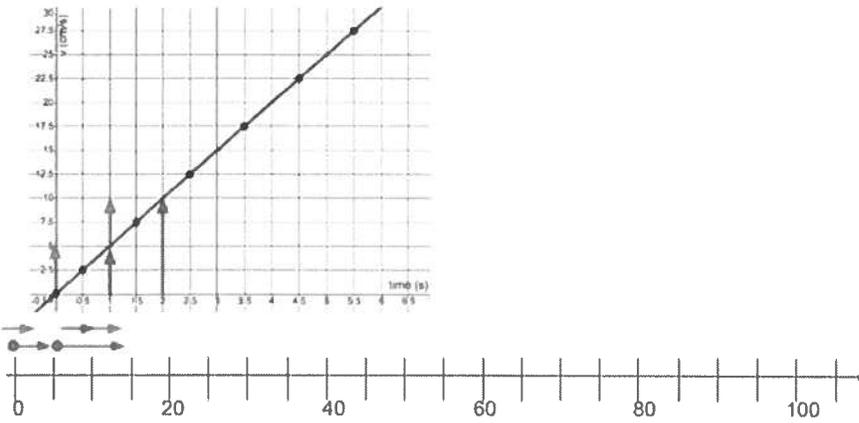
- At  $t = 0$  s, the length of the velocity vector is 0 . . .

. . . but the acceleration (change in velocity over a second) is 0.5 units, so if I add 0.5 to zero, I get the length of the next velocity vector, 0.5 units . . .

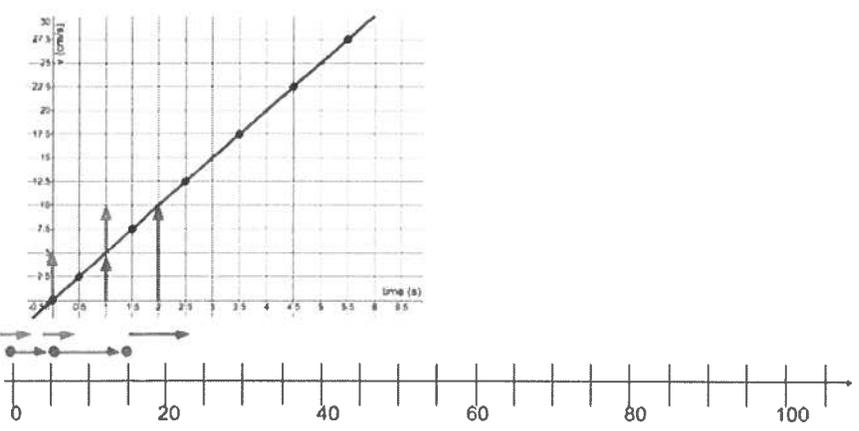


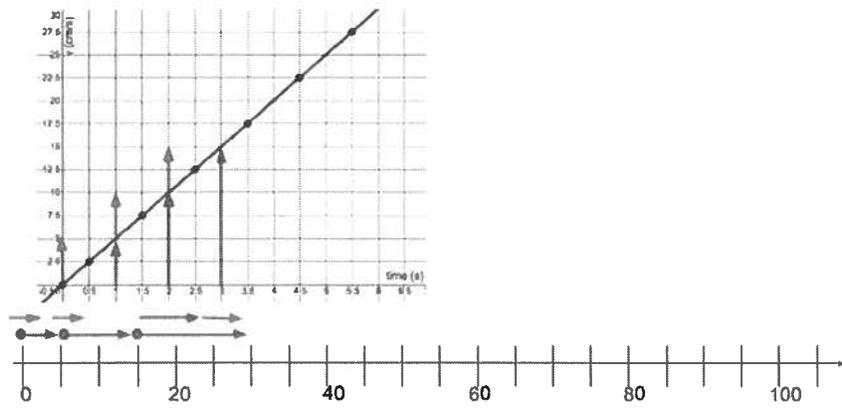
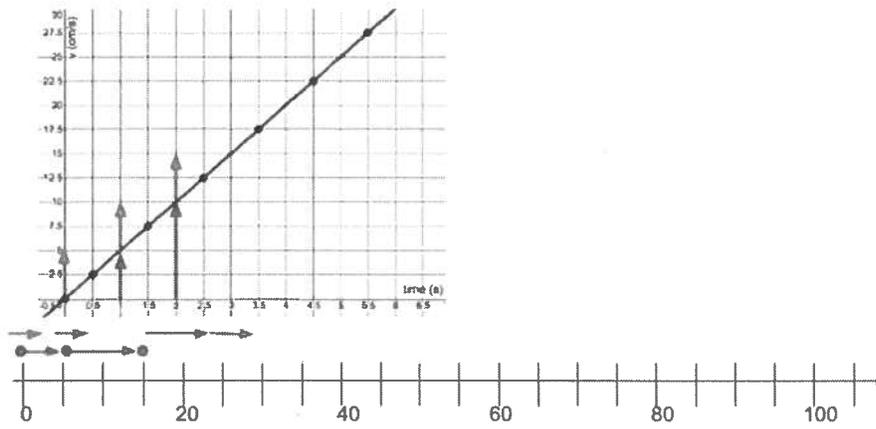
If I add the acceleration (still 0.5 units, because acceleration is constant) to the average velocity in the first second, I get the average velocity over the next second, 1.0 units . . .



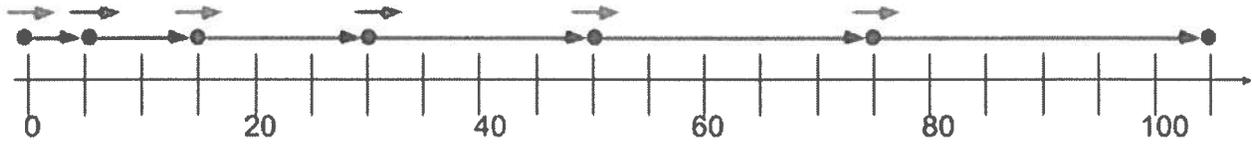


... repeat for the next second

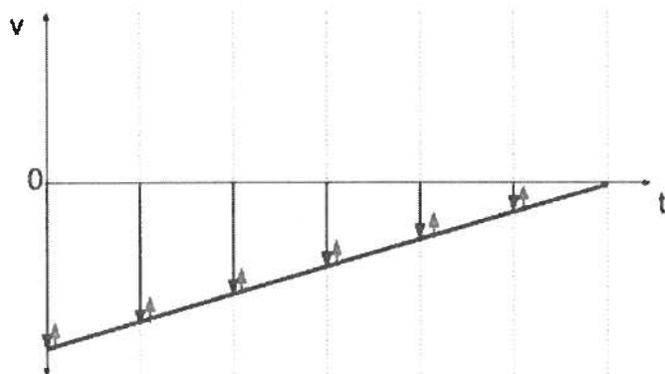




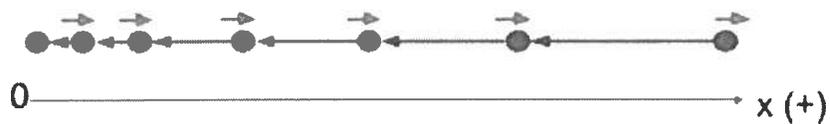
By the time I have done this for all seconds, I'd have . . .



This strategy works with all kinds of motions. Let's try one where the object is going backwards and slowing down.



In this case the velocity vectors are pointing in the negative direction and getting shorter. The acceleration vectors are pointing in the positive direction.

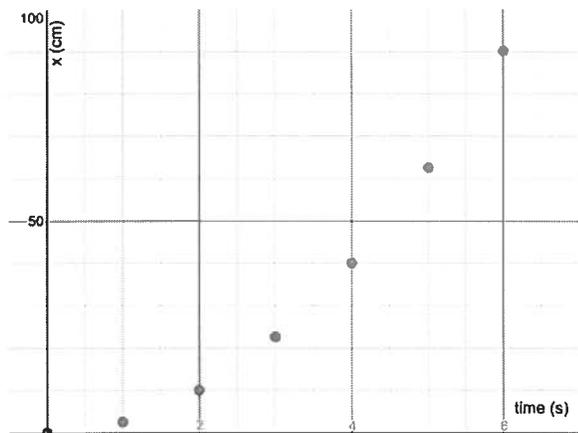


Check your understanding now by clicking on the check mark:

## Enhanced Equations for Describing Motion

Remember the original data and x vs. t graph for our toy car?

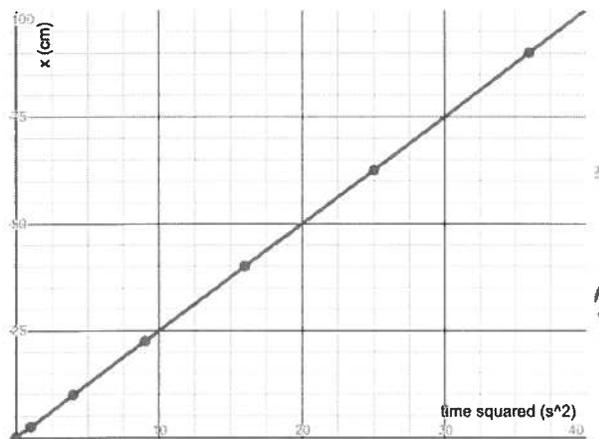
time (s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
x (cm)	0.0	2.5	10.0	22.5	40.0	62.5	90.0



When we were learning how to build mathematical models, we saw that a graph that has this shape can be linearized by squaring the horizontal axis values and re-plotting that data. Let's try it and see what we can learn.

time (s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
time <sup>2</sup> (s <sup>2</sup> )	0.0	1.0	4.0	9.0	16	25	36
x (cm)	0.0	2.5	10.0	22.5	40.0	62.5	90.0

Not surprisingly, this gives us a nice straight line.



$$y_1 \sim mx_1 + b$$

STATISTICS

$$r^2 = 1$$

$$r = 1$$

PARAMETERS

$$m = 2.5$$

RESIDUALS

$e_1$  plot

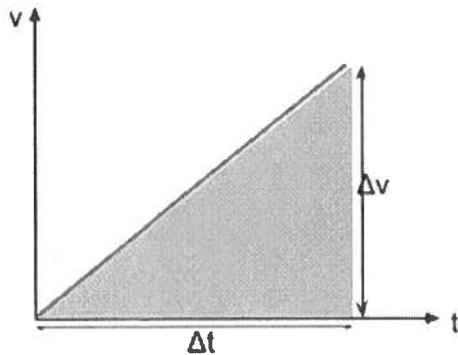
$$b = 0$$

$$x = 2.5 \frac{\text{cm}}{\text{s}^2} \cdot t^2$$

The equation of this line is . . .

If you're really observant, you might notice that this is half of the value of the acceleration we found earlier. This turns out not to be a coincidence.

We can explore this by thinking about a velocity vs. time graph for an object that speeds up from 0 by some change in velocity over some change in time:



The displacement during this time will be the area under the graph, which is  $\Delta x = \frac{\Delta v \cdot \Delta t}{2}$

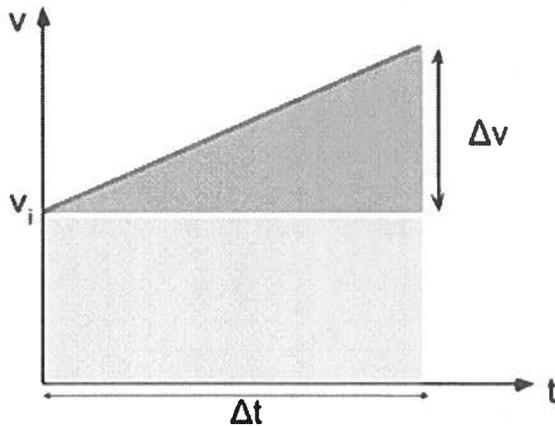
But we can use the definition of acceleration to replace  $\Delta v$  in this equation . . .

$$a \equiv \frac{\Delta v}{\Delta t} \text{ But if both sides are multiplied by } \Delta t, \text{ we get } \dots a\Delta t = \Delta v$$

So when we substitute, we get 
$$\Delta x = \frac{(a\Delta t) \Delta t}{2} = \frac{a\Delta t^2}{2} = \frac{a}{2}\Delta t^2$$

If the starting time was 0 and the starting position was also 0, the changes in x and t are just the current values of x and t, so this explains why the slope of the x vs. t squared line was half the acceleration.

To make our equation work not just when the starting velocity is zero, let's draw our graph for when the object starts with a velocity we'll call  $v_i$  (the "i" stands for "initial" which means "starting").



We know the displacement represented by the triangular part of the area already. Now let's add the rectangular part, which is also under the graph and so it's part of the displacement . . .

$$\Delta x = v_i \Delta t + \frac{a}{2} \Delta t^2$$
 This is one version of a very powerful equation called the "time-dependent kinematics equation".

Another way of writing it expresses the position, instead of the displacement by adding the starting position to the displacement on the right hand side of the equation:

$$x = x_i + v_i \Delta t + \frac{a}{2} \Delta t^2$$



Check your understanding now by clicking on the check mark:

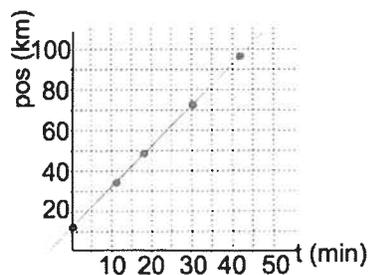
## Study Guide for . . .

### Constant Acceleration Particle Model: A more advanced model of motion in physics.

#### Introduction:

- What is one of the of the main focuses in physics?
  - How is the motion we're studying in this unit going to be different than in the last unit (CVPM)?
  - How is the motion we're studying in this unit going to be the same as in the last unit?
- 

#### Motion Maps:



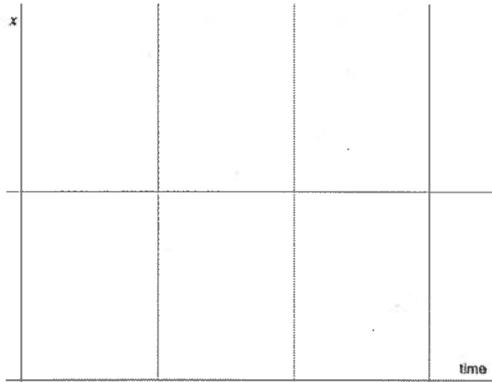
- List two features of a motion map for the motion represented by the position vs. time graph above:
  - 
  -
- If the motion was not constant speed, but involved an object speeding up, how would the motion map be different?



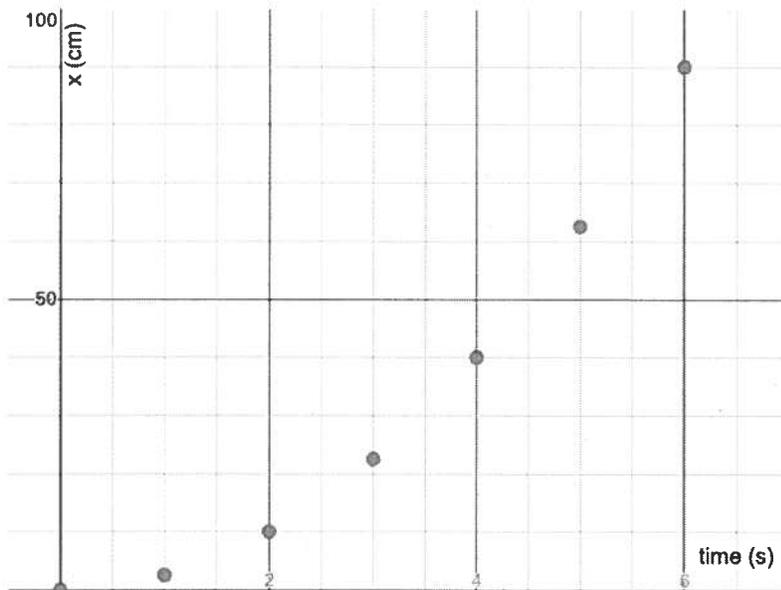
List any questions you have at the end of this introduction. These might be things you should go back to later to see if you've learned them. Or maybe you should talk to your teacher or a classmate for clarification.

### Position vs. time graphs

- Sketch the general shape of the position ( $x$ ) vs. time ( $t$ ) graph for an object that is going forward and speeding up from a stop.



- When you studied model-building back in the first unit (STES), what did you learn was a good way to modify the data when the graph looked like this?



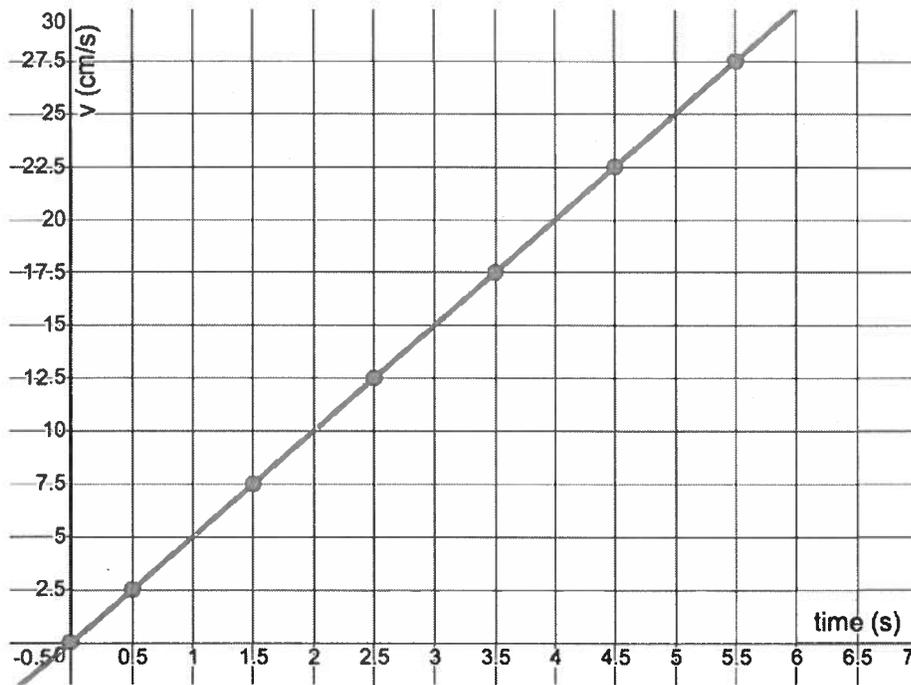
- On the  $x$  vs.  $t$  graph above for a toy car rolling down a ramp, sketch a line to show how you might find the average velocity over the first 6 seconds.
- What is that average velocity?
  
- Now, sketch the line and calculate the velocity during the first 2 seconds:

- Now the next two seconds (2 s  $\rightarrow$  4 s):
- Now the last two seconds (4 s  $\rightarrow$  6 s):
- What is happening to the average velocity over these two second intervals as the starting time of the interval increases?
- Over which of the three 2-second intervals is the average velocity the same as the overall average velocity for the whole 6 seconds?
- What is a tangent line and what does its slope tell us about the velocity of the object?
- During a period when the velocity is changing steadily, at which exact moment in time is the instantaneous velocity the same as the average velocity?

 List any questions or thoughts you have at the end of this section. These might be things you should go back to later to see if you've learned them. Or maybe you should talk to your teacher or a classmate for clarification.

## Velocity vs. time graphs

- What did we learn from our last section that allows us to make a precise graph of velocity vs. time for the car?



- What is the special name for the slope of a velocity vs. time graph?
- Show how you can calculate this slope, including units in your calculation.
- What is the physical meaning of this slope?
- What is the physical meaning of the vertical intercept of this graph?
- Write the full equation for this particular graph:
- What does CAPM stand for?
- Now write the general equation for any graph of velocity vs. time when the motion is CAPM:

- In the table below, identify the descriptions of motion that match the v vs. t graphs by writing “forward” or “back” in the second column and “speeding up”, “slowing down” or “constant speed” in the third column

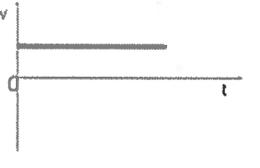
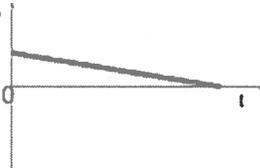
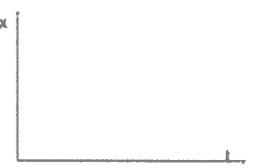
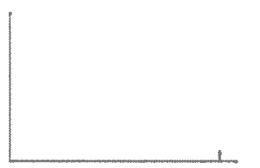
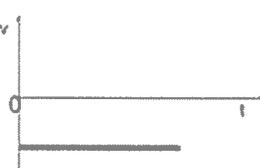
v vs. t graph	Direction of motion	Change in motion
		
		
		
		
		
		



List any questions or thoughts you have at the end of this section. These might be things you should go back to later to see if you've learned them. Or maybe you should talk to your teacher or a classmate for clarification.

**Putting x and v vs. t graphs together:**

- What is involved in doing the pencil test?
- What two things does the slope of the pencil tell us?
  - 
  -
- In the table below, for each velocity vs. time graph, sketch the position vs. time graph that shows the same motion.

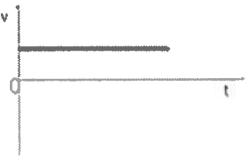
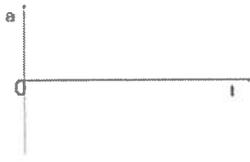
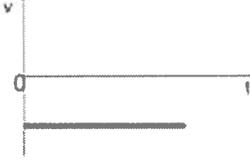
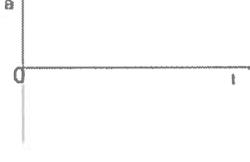
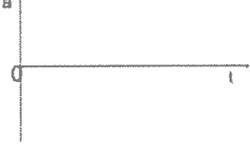


List any questions or thoughts you have at the end of this section. These might be things you should go back to later to see if you've learned them. Or maybe you should talk to your teacher or a classmate for clarification.

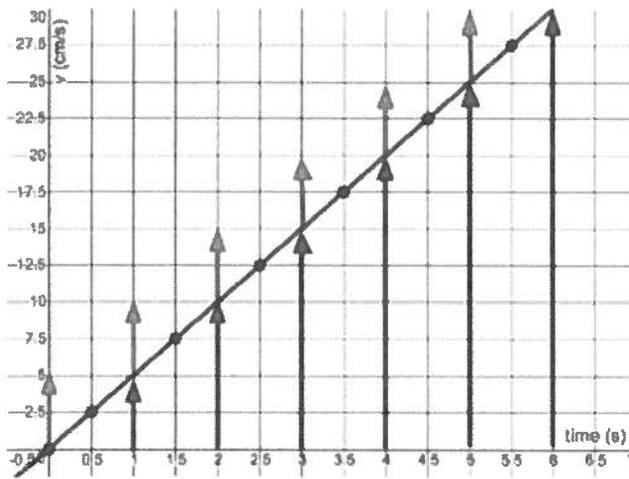
**Adding acceleration to our graphical models**

**Part I: a vs. t graphs:**

- Define acceleration in terms of . . .
  - What it physically means:
  - How it's modeled on a v vs. t graph:
- In the table below for each velocity vs. time graph, sketch the position vs. time graph that shows the same motion.

## Part II: Enhancing Motion Maps



- In the graph above, what does it mean that the blue arrows are pointing in the positive direction?
- What does it mean that the blue arrows are getting longer with increasing time?
- What is the meaning of the **length** of the red arrows?
- What is the meaning of the **direction** of the red arrows?
- Fill in the blanks in the following statements:
  - If the object is speeding up, the acceleration vectors are in the \_\_\_\_\_ direction as the velocity vectors.
  - If the object is slowing down, the acceleration vectors are in the \_\_\_\_\_ direction as the velocity vectors.

- In the following table, create motion maps that match each of the descriptions.

The object is moving forward and speeding up.



The object is moving forward and slowing down.



The object moves forward, but slows to a stop, then turns and moves backward while speeding up.



The object moves backward while slowing down.



-  List any questions or thoughts you have at the end of this section. These might be things you should go back to later to see if you've learned them. Or maybe you should talk to your teacher or a classmate for clarification.

## **Enhanced Equations for Describing Motion**

- What is the general time-dependent kinematics equation?
  - At  $t = 0$  s, an object is at a position of 5 m and is moving forward with a speed of 50 m/s, but it was slowing down at a rate of 10 m/s/s, what would be the specific time-dependent kinematics equation that could predict the position of this object at any time?
  
- What is the general time-independent kinematics equation?
  - For what situations would this equation be most useful?

### CAPM Idea Builder #1

The data in this table are for a wheel rolling from rest down a ramp.

time (s)	0	1.0	2.0	3.0	4.0	5.0	6.0
x (cm)	0.0	5.0	20.0	45.0	80.0	125.0	180.0

- Plot the position vs. time graph for this motion:

